

**Department of Mathematics**  
**Central University of Kashmir Srinagar.**

**Course Structure Aand Syllabus for M.A/M.sc Mathematics**

S.No	Course Code	Course Title	Credits	Marks(100)	
	<b>Semester I</b>			CIA	ESE
01	MMT-C101	Algebra I	4	40	60
02	MMT-C102	Real Analysis I	4	40	60
03	MMT-C103	Complex Analysis I	4	40	60
04	MMT-C104	Theory of Numbers I	4	40	60
05	SS	Soft Skills Elective	4	40	60
	<b>Semester II</b>				
06	MMT-C201	Algebra II	4	40	60
07	MMT-C202	Real Analysis II	4	40	60
08	MMT-C203	Topology	4	40	60
09	MMT-C204	Differential Geometry	4	40	60
10	SO	Social Orientation Elective	4	40	60
	<b>Semester III</b>				
	<b>Core Courses</b>				
11	MMT-C301	Theory of Rings and Modules	4	40	60
12	MMT-C302	Functional Analysis I	4	40	60
13	MMT-C303	Complex Analysis II			
	<b>Elective Courses</b>				
14	MMT-E301	Probability and Statistics	4	40	60
15	MMT-E302	Measure and Integration	4	40	60
16	MMT-E303	Descrete Mathematics	4	40	60
17	MMT-E304	Theory of Numbers II	4	40	60
18	MMT-E305	Semigroups and Non Commutative rings	4	40	60
	<b>Semester IV</b>				
	<b>Core Courses</b>				
19	MMT-C401	Linear Algebra	4	40	60
20	MMT-C402	Functiuonal Analysis II	4	40	60
21	MMT-C403	Ordinary and Partial Differential Equations	4	40	60
	<b>Elective Courses</b>				
22	MMT-E401	Graph Theory	4	40	60
23	MMT-E402	Probability and Measure	4	40	60
24	MMT-E403	Operations Research	4	40	60
25	MMT-E404	Commutative Algebra	4	40	60
26	MMT-E405	Category Theory and Infinite Abelian Groups	4	40	60

## Algebra I (MMT-C101)

### Unit-I

Review of basic concepts of groups and subgroups, cosets and their properties. Normal subgroups and quotient groups. Group homomorphisms and Isomorphisms. Cauchy's and Sylow's Theorems for abelian groups. Automorphisms, Cayley's Theorem and its applications. Permutation groups, Simple groups.

### Unit-II

Conjugacy, Normalizer and Center of a group. Class equation of finite group and its applications. Cauchy's Theorem, Sylow's Theorem and their applications. Direct products, Finite abelian groups and Fundamental Theorem for finite abelian groups. Solvable groups, Nilpotent groups and Jordan-Hölder Theorem.

### Unit-III

Review of basic concepts of Rings, sub rings, Integral domains and Fields. Ring homomorphisms and quotient rings. Ideals and their properties. Rings and fields of fractions and embedding Theorems. The Chinese Remainder Theorem. Euclidean domains, Principal Ideal domains; examples and properties.

### Unit-IV

Unique factorization domains; examples and properties, factorization in Gaussian Integers. Polynomial rings, Polynomial rings over a field, Gauss's Lemma, Polynomial rings that are unique factorization domains. Irreducibility criteria, Hilbert's basis Theorem.

### Text Books:

1. I. N. Herstein, **Topics in Algebra**, 2nd Edition Wiley (Section 2.1-Section 2.14).
2. N.S. Gopalakrishnan, **University Algebra**. (Section 1.13-1.14).
3. D.S. Dumit and R.M. Foote, **Abstract Algebra**, 3rd Edition Wiley (Section 7.1-Section 9.6).

### Reference Books:

1. Joseph A. Gallian **Contemporary Abstract Algebra**, 4th Edition Narosa.
2. M. Artin, **Algebra**, Prentice Hall, 1991.
3. J.B. Fraleigh, **A First Course in Abstract Algebra**, Addison-Wesley, 2002.
4. N. Jacobson, **Basic Algebra- I**, Hemisphere Publishing Corporation, 1984
5. S. Lang, **Algebra**, Springer, 2002.

## Real Analysis I (MMT-C102)

### Unit-I

A review of basic set theory, finite, countable and un-countable sets, real number system as complete ordered field, Archimedean property, Bounded and un-bounded sets, Supremum and Infimum, Dedekend's form of completeness property. Definition and existence of the Riemann-Stieltjes integral, upper and lower sums and integrals. Refinement of partitions. Necessary and sufficient conditions for R-S integrability. Some properties of the Riemann- Stieltjes integrals. The integral as a limit of a sum. R-S integrability of continuous and monotonic functions, Reduction of the R-S integral to a Riemann integral. First and second Mean Value Theorems. Change of variables.

### Unit-II

Improper Integrals: Integration of un-bounded functions with finite limit of integration. Comparison tests for convergence of improper integrals. Cauchy's test for convergence. Absolute convergence. Infinite range of integration of bounded functions . Convergence of integrals of unbounded functions with infinite limits of integration. Integration as a product of functions. Abel's and Dirichlet's tests of convergence. Fourier Series: Euler Fourier Formula, piecewise monotonic and piecewise continuous functions, periodic functions. Fourier series expansion of a function  $f(x)$  in the intervals  $(-\pi, \pi)$ ,  $(-c, c)$ ,  $(a, b)$ . Fourier series for even and odd functions. Fourier sine series expansion and Fourier cosine series expansion of  $f(x)$  in the intervals  $(0, \pi)$ ,  $(0, c)$ . Some theorems : if  $f(x)$  is bounded and integrable function on  $(-\pi, \pi)$  and if  $a_n, b_n$  are its Fourier coefficients, then  $\sum_{n=0}^{\infty}(a_n^2 + b_n^2)$  converges. Riemann-Lebesgue theorem.

### Unit-III

Uniform convergence of sequences and series of functions : Point wise convergence, uniform convergence on an interval, Cauchy's criterion for uniform convergence,  $M_n$  test for uniform convergence of sequences, Weirestrass's M-Test, Abel's and Dirichlet's tests for uniform convergence of series. Uniform convergences and continuity, uniform convergence and integration and uniform convergence and differentiation, Weirestrass's Approximation Theorem .

### Unit-IV

Functions of bounded variation and Rectifiable curves: Concave and convex functions, Properties of monotonic functions, Functions of bounded variations, Total variation, Additive property of total variation , Functions of bounded variation expressed as a difference of increasing functions, Continuous functions of bounded variation. Curves and paths, Rectifiable paths and arc length, Additive and continuity property of arc length, Change of parameter.

### Text Books:

1. Walter Rudin, **Principles of Mathematical Analysis**, 3rd Edition. Mc-Graw Hill, 1976.

2. T. M. Apostol, "Mathematical Analysis-Second Edition, Narosa Publishing House.
3. S.C. Malik, **Mathematical Analysis** Wiley Eastern Limited Urvashi, Press, 1983, Meerut

**Reference Books:**

1. A.J. White, **Real Analysis- An Introduction**, Addison, Wesley, 1968
2. S. Lang, **Real Analysis**, Addison, Wesley, 1969
3. R. Goldberg, **Methods of Real Analysis**, John Wiley and Sons, 1976
4. T.M. Apostol, **Mathematical Analysis**, Narosa, 2004
5. H.L. Royden, **Real Analysis**, MacMillan, 1988
6. G.B. Folland, **Real Analysis**, Brooks/Cole, 1992

## Complex Analysis I (MMT-C103)

### Unit-I

Continuity and Differentiability of Complex Functions. Analytic Functions and C-R equations. Necessary and Sufficient condition for a function to be Analytic. Harmonic Functions. Complex Integration. Cauchy's Theorem, Cauchy-Goursat Theorem. Index of a point with respect to a curve. Cauchy's Integral formula for Analytic function and its Derivatives. Morera's Theorem and Cauchy's Inequalities. Entire Functions and Liouville's Theorem.

### Unit-II

Möbius Transformations, their properties and classifications. Fixed points, Inverse points and Critical points of Möbius Transformations. Cross Ratio and its Preservation under Möbius Transformation. Bilinear Transformation carrying i) Circles to Circles, ii) Upper half Plane onto a unit Circle, iii) Right half Plane into a unit Circle, iv) Unit circle onto a Unit circle and v) Arbitrary Circle to Arbitrary circle. The Transformations  $w = \sqrt{z}$ ,  $w = z^2$  and  $w = \frac{1}{2}(z + \frac{1}{z})$ . Conformal Mapping. Necessary and sufficient Condition for a Mapping to be Conformal.

### Unit-III

Power Series. Cauchy-Hadamard Formula for the radius of Convergence of power series. Absolute convergence. Taylor's theorem. Expansion of Analytic Function in a Power Series. Binomial Theorem for any Index. Hadamard-Phragmén Theorem. Picard's Theorem. Laurant's Theorem and its Consequences. Laurant Series. Classification of Singularities. Isolated singularity. Properties of Removable Singularity, poles and Essential singularity. Riemann's Theorem. Cassoriti-Wierstrass Theorem on Essential Singularity.

### Unit-IV

Calculus of Residues. Cauchy-Residue Theorem. Jordan's Lemma. Evaluation of Integrals by the method of Residues. Parseval's Identity. Infinite products and their convergence. Necessary and sufficient Conditions for the Convergence of Infinite Products. Equivalence between Series and Infinite Products. Necessary and sufficient Conditions for the Absolute Convergence of Infinite Products. If  $\sum a_n$  and  $\sum a_n^2$  are convergent, then so is  $\prod (1 + a_n)$ .

### Text Books:

1. S. PONNUSAMY, **Foundations of Complex Analysis**.
2. Richard A Silverman, **Introductory Complex Analysis**, Prentice Hall, Inc., 1967.
3. J.B. Conway, **Functions of One Complex Variable, II**, Graduate Text in Mathematics, 159, Springer-Verlag, 1995.
4. L.V. Ahlfors, **Complex Analysis**, 3rd Edition, MC Graw Hill, New York, 1979.

5. Nihari Z, **Conformal Mappings**

**Reference Books:**

1. E.C. Titchmarsh, **Theory of Functions**, Oxford University Press.
2. W.H.J. Fuchs, **Topics in Theory of Functions on One Complex Variable**.
3. E.Hille, **Analytic Function Theory**, Vol. 1, Ginn, 1959.
4. R. Nevanlinna, **Analytic Functions**, Springer 1970.
5. M.R. Spiegel, **Theory and Problems of Complex Variables**, Schaums Outline Series, Mc Graw Hill, New York , 1985.
6. R.V. Churchill, J.W. Brown and R.F. Verkey, **Complex Variables and Applications**, 5, Ed. MC- Graw Hill, New York, 1989.

## Number Theory I (MMT-C104)

### Unit-I

Divisibility, the Division Algorithm. Greatest Common Divisor and its properties, Least Common Multiple. Prime numbers, Euclid's first Theorem, factorization in primes, Fundamental Theorem of Arithmetic, Linear Diophantine Equations; Necessary and Sufficient condition for solvability of linear Diophantine equations. Sequences of Primes : Euclid's second theorem, infinitude of the primes of the form  $4n+3$  and  $6n+5$ , No polynomial with integral coefficients, not a constant, can represent primes for all integral values. Fermat numbers and properties, Mersenne numbers, Arbitrary large gaps in the sequence of primes.

### Unit-II

Congruences, Complete Residue Systems and Reduced Residue Systems and their properties, Euler's  $\phi$  Function,  $\phi(mn) = \phi(m)\phi(n)$ ,  $(m, n) = 1$ . Fermat's theorem and Euler's Theorem. Wilson's Theorem and its application to the solution of the congruence  $x^2 = -1 \pmod{p}$ , Solution of linear congruences, The necessary and sufficient for the solution of the congruence  $a_1x_1 + a_2x_2 + \dots + a_nx_n = c \pmod{m}$ . Chinese Remainder Theorem, Congruences of higher degree.

### Unit-III

Polynomial congruence  $f(x) = 0 \pmod{m}$ . Congruences with prime power moduli. Lagrange's theorem; the polynomial congruence  $f(x) = 0 \pmod{m}$  has at most  $n$  solutions, The criteria for the congruence  $f(x) = 0 \pmod{m}$  of degree  $n$  having exactly  $n$  solutions. Factor Theorem and its generalization, Polynomial congruence  $F(x_1, x_2, \dots, x_n) = 0 \pmod{m}$  in several variables, Equivalence of polynomial congruences, number of solutions in polynomial congruences, Chevalley's Theorem, Warning's Theorem.

### Unit-IV

Quadratic form over the field of characteristic  $\neq 2$ ; Equivalence of Quadratic forms Witt's Theorem, Representation of field elements, Hermite's Theorem on the minima of a positive definite quadratic form and its application to the sum of two squares. Integers belonging to a given exponent  $\pmod{p}$  and related results. Converse of Fermat's Theorem. If  $d|(p-1)$ , the Congruence  $x^d = 1 \pmod{p}$  has exactly  $d$ -solutions. If any integer belongs to  $t \pmod{p}$ , then exactly  $\phi(t)$  incongruent numbers belong to  $t \pmod{p}$ . Primitive roots. Existence of  $\phi(p-1)$  primitive roots for a given odd prime  $p$ . The function  $\lambda(m)$  and its properties. Primitive root of  $m$ . The number of primitive roots of  $1, 2, 4, p^\alpha$  and  $2p^\alpha$ .

### Text Books:

1. Ivan Niven and H.S. Zuckerman, **An Introduction to the Theory of Numbers**, 5th edition, Wiley Eastern, 2000.
2. Leveque, **Topics in Number Theory**, Springer.
3. Boevich and Shaferwich, **Number theory**, I.R. Academic Press.

4. T.M. Apostol **Theory of Numbers**, Springer

**Reference Books:**

1. E. Landau **Elementary Number theory** AMS Chelsea publishing
2. J. P. Serre **A course in Arithmetic** Vol Springer
3. D.M. Burton, **Elementary Number Theory**, 2nd edition, Universal Book Stall, New Delhi, 1994.
4. Y. Hsiung, **Elementary Theory of Numbers**, World Scientific, 1992; First Indian Reprint, Allied Publishers Limited, 1995.
5. G.H. Hardy and E.M. Wright, **An Introduction to the Theory of Numbers**, 4th edition, Oxford, Clarendon Press, 1960.
6. G.E. Andrews, **Number Theory**, Hindustan Publishing Corporation, New Delhi, 1992.
7. S.G. Telang, **Number Theory**, Tata McGraw Hill Publishing Company Limited, New Delhi, 1996



## Algebra II (MMT-C201)

### Unit-I

Systems of linear equations, Matrices and Elementary Row operations, Row-reduced Echelon matrices, Matrix multiplication and Invertible matrices. Vector spaces, subspaces, Bases and Dimensions, coordinates, Row-equivalence and computations concerning subspaces. Linear Transformations, The Algebra of linear transformations, Representation of transformations by matrices, Linear Functionals, The double Dual and the Transpose of a linear transformation.

### Unit-II

Characteristic of a field, Prime subfield of a field. Field Extensions, Simple extensions, Algebraic, transcendental and finite extensions. Classical Straightedge and Compass constructions.

### Unit-III

Splitting fields, Uniqueness of Splitting fields. The Algebraic Closure of a field. The Fundamental Theorem of Algebra. Separable and Inseparable extensions. Perfect fields, Cyclotomic polynomials and extensions.

### Unit-IV

Automorphism group of a field, Fixed field of a field; their properties and examples. Galois extensions, Galois groups. Fundamental Theorem of Galois Theory and examples on this theorem. Finite fields and their properties.

### Text Books:

1. Kenneth Hoffman and Ray Kunze, **Linear Algebra**, 2nd Edition Prentice Hall (Ch. 1- Ch. 3).
2. D.S. Dumit and R.M. Foote, **Abstract Algebra**, 3rd Edition Wiley.

### Reference Books:

1. I. N. Herstein, **Topics in Algebra**, 2nd Edition Wiley (Section 2.1-Section 2.14).
2. Joseph A. Gallian **Contemporary Abstract Algebra**, 4th Edition Narosa.
3. M. Artin, **Algebra**, Prentice Hall, 1991.
4. J.J. Rotman, **Galois Theory**, Graduate Texts in Mathematics, Springer.

## Real Analysis II (MMT-C202)

### Unit-I

Multivariable Differential Calculus: The directional derivative, The total derivative, The total derivative expressed in terms of partial derivatives, The matrix of linear transformation, The Jacobian matrix, The Chain rule, The matrix form of chain rule, The mean value Theorem for differentiable functions, A sufficient condition for differentiability, A sufficient condition for equality of mixed partial derivatives, Taylor's Theorem for function from  $R^n$  to  $R$ .

### Unit-II

Implicit Functions and Extremum Problems: Functions with non-zero Jacobian determinant, The inverse function Theorem, The implicit function Theorem, Extrema of real valued functions of one variable, Extrema of real valued functions of several variables, Extremum problem with side conditions.

### Unit-III

Measurable sets : Definitions of outer and inner measure of a set, some basic properties. Outer measure of an interval as its length, measurability of the union of two measurable sets, measurability of the countable union of pair wise dis-joint bounded measurable sets, inequality concerning countable additivity of an outer measure. Borel measurable sets, sets of measure zero and non-measurable sets. Measure of the outer and inner limiting sets, measurable functions and their structures

### Unit-IV

Riemann integral and its deficiency, Lebesgue integral of bounded function, comparison of Riemann and Lebesgue integrals, properties of Lebesgue integral for bounded measurable function, the Lebesgue integral for unbounded functions, integral of non-negative measurable functions, general Lebesgue integral, improper integral.

### Text Books:

1. H.L. Royden, **Real Analysis**, MacMillan, 1988
2. Walter Rudin, **Principles of Mathematical Analysis**, 3rd Edition. Mc-Graw Hill, 1976.
3. T. M. Apostol, **Mathematical Analysis-Second Edition**, Narosa Publishing House.
4. D.V. Widder, **Advanced Calculus**. 2/e, Prentice Hall of India, New Delhi

### Reference Books:

1. R. Goldberg, **Methods of Real Analysis**, John Wiley and Sons, 1976
2. T.M. Apostol, **Mathematical Analysis**, Narosa, 2004

3. G.De Barra, **Measure Theory and Integration** Narosa Publishing House, New Delhi

## Topology (MMT-C203)

### Unit-I

Review of countable and uncountable sets, Schroeder-Bernstein theorem, Axiom of Choice and its various equivalent forms, Definition and examples of metric spaces, Open and Closed sets, completeness in metric spaces, Baires Category theorem, and applications to the (i) Non-existence of a function which is continuous precisely at irrationals (ii) Impossibility of approximating the characteristic of rationals on  $[0, 1]$  by a sequence of continuous functions.

### Unit-II

Completion of a metric space, Cantor's intersection theorem with examples to demonstrate that each of the conditions in the theorem is essential, Uniformly continuous mappings with examples and counter examples, Extending Uniformly continuous maps, Banach's contraction principle.

### Unit-III

Topological spaces; Definition and examples, elementary properties, Kuratowskis axioms, continuous mappings and their characterizations, pasting Lemma, Nets in topological spaces, convergence of nets and continuity in terms of nets, Bases and sub-bases for a topology, Lower limit topology, concepts of first countability, second countability, separability and their relationships, counterexamples and behavior under subspaces, weak topology generated by a family of mappings, product topology.

### Unit-IV

Compactness and its various characterizations, Heine-Borel theorem, compactness, sequential compactness and total boundedness in metric spaces. Lebesgues covering lemma, continuous maps on a compact space. Tychonoffs product theorem, Separation axioms  $T_i$  ( $i=1,2,3,1/2,4$ ) and their permanence properties, connectedness, local connectedness, their relationship and basic properties, Connected sets in  $\mathbb{R}$ . Urysohns lemma. Urysohns metrization theorem. Tietzes extension theorem, one point compactification.

### Text Books:

1. G.F.Simmons : **Introduction to topology and Modern Analysis**
2. J. Munkres : **Topology, A First Course.**
3. K.D. Joshi : **Introduction to General topology**
4. J.L.Kelley : **General topology**
5. Murdeshwar ; **General topology**
6. S.T. Hu : **Introduction to General topology.**

### Reference Books:

1. Dugundji J, **Topology**, Prentice Hall of India, 1966 .
2. Willard, **General Topology**, Addison-Wesley 1970.
3. I.M. Singer and J.A. Thorpe, **Lecture Notes on Elementary Topology and Geometry**, Undergraduate Texts in Mathematics, Springer-Verlag, 1976.

## Differential Geometry (MMT-C204)

### Unit-I

Curves : Differentiable curves, Regular point, Parameterization of curves, arc-length, and arc-length is independent of parameterization, unit speed curves. Plane curves: Curvature of plane curves, osculating circle, centre of curvature. Computation of curvature of plane curves. Directed curvature, fundamental theorem for plane curves. Examples: Straight line, circle, ellipse, tractrix, evolutes and involutes. Space curves: Tangent vector, unit normal vector and unit binormal vector to a space curve. Curvature and torsion of a space curve. The Frenet-Serret theorem. First Fundamental theorem of space curves. Intrinsic equation of a curve. Computation of curvature and torsion. Characterization of Helices and curves on sphere in terms of their curvature and torsion. Evolutes and involutes of space curves.

### Unit-II

Surfaces and Regular surfaces with examples, coordinate charts or curvilinear coordinates, change of coordinates. Critical Value, Regular value and Regular surface. Tangent plane at a regular point, normal to the surface, orient able surface, differentiable mapping between regular surfaces and their differential. Fundamental form or a metric of a surface, line element, invariance of a line element under change of coordinates, angle between two curves, condition of orthogonality of coordinate curves. Area of bounded region, invariance of area under change of coordinates.

### Unit-III

Curvature of a Surface: Normal curvature, Eulers work on principal curvature,. Qualitative behaviour of a surface near a point with prescribed principal curvatures. The Gauss map and its differential. The differential of Gauss is self-adjoint. Second fundamental form. Normal curvature in terms of second fundamental form. Meunier theorem. Gaussian curvature, Weingarten equation. Gaussian curvature  $K(p) = (eg-f^2)/EG-F^2$  .surface of revolution. Surfaces with constant positive or negative Gaussian curvature. Gaussian curvature in terms of area. Line of curvature, Rodrigues formula for line of curvature, Equivalence of Surfaces: Isometry between surfaces, local isometry, and characterization of local isometry.

### Unit-IV

Christoffel symbols. Expressing Christoffel symbols in terms of metric coefficients and their derivative. Theorema egregium (Gaussian curvature is intrinsic). Isometric surfaces have same Gaussian curvatures at corresponding points. Gauss equations and Manardi Codazzi equations for surfaces. Fundamental theorem for regular surface. (Statement only).

Geodesics: Geodesic curvature, Geodesic curvature is intrinsic, Equations of Geodesic, Geodesic on sphere and pseudo sphere. Geodesic as distance minimizing curves. Gauss-Bonnet theorem (statement only).Geodesic triangle on sphere. Implication of Gauss-Bonnet theorem for Geodesic triangle.

**Text Books:**

1. John Mc Cleary: Geometry from a differentiable Viewpoint. (Cambridge Univ. Press).
2. Andrew Pressly, Elementary Differential Geometry (Springer Verlag, UTM).
3. Barret O'Neil, Elementary Differential Geometry, Academic Press (2006).
4. C.Baer, Elementary Differential Geometry, Cambridge Univ. Press (2010).

**Reference Books:**

1. W. Klingenberg: A course in Differential Geometry (Spring Verlag)
2. J. M. Lee : Riemannian Manifolds, An Introduction to Curvature (Springer Verlag).

## **Theory of Rings and Modules (MMT-C301)**

### **Unit-I**

Definition and simple properties of rings, sums direct sums and products of ideals. Complete matrix rings and ideals in complete matrix rings. Direct and subdirect sums of rings, subdirectly irreducible rings, Boolean rings. Prime radical and prime rings.

### **Unit-II**

Rings of endomorphisms, irreducible rings of endomorphisms, R-modules and rings of endomorphisms. Irreducible rings and vector spaces, dense rings of linear transformations, subspaces and descending chain conditions. The WedderburnArtin Theorem.

### **Unit-III**

Modules and submodules, sum and intersection of submodules, Linear combinations and spanning sets, Homomorphisms, Isomorphism Theorems, Inverse image of submodules, Annihilator, Order of elements.

### **Unit-IV**

Direct summands, split homomorphisms, projections, idempotent endomorphisms, essential and superfluous submodules, semi-simple modules, socle and radical of modules, basis and rank free modules.

### **Text Books:**

1. Neal H. McCoy: **Theory of Rings.**
2. Anderson and Fuller: **Rings and Categories of Modules**

### **Reference Books:**

1. Joachim Lambek **Lectures on Rings and Modules.** AMS Chelsea Publishing.
2. T. Y Lam **Lectures on Rings and Modules.** Graduate Texts in Mathematics, springer.



## Functional Analysis I (MMT-C302)

### Unit-I

Normed Linear Spaces and Banach spaces : Definition and examples, Continuous Linear Transformation and their Characterization, Completeness of the Space  $B(X, Y)$  of Bounded Linear Operators, Isometric Isomorphism, Dual of a Normed Linear Space, Computing the Dual of well known Banach Spaces, Equivalence of Norms of Finite Dimensional space, Hahn Banach Theorem and its Applications.

### Unit-II

The Natural Embedding of a Normed Linear Space  $N$  in  $N^{**}$ , Reflexive Normed Linear Spaces, weak and Weak \* Topologies, characterization of Reflexive Banach spaces, Open Mapping Theorem, Projection on a Banach Space, Closed Graph Theorem and Banach Steinhaus Theorem (Uniform Boundedness Principle), Conjugate of a Continuous Linear Operator and its Properties.

### Unit-III

Definition and examples of Hilbert Spaces, Cauchy's Schwartz Inequality and Parallelogram Law, Orthogonal Complements, Orthogonal Decomposition of Hilbert Space, Orthonormal Systems, Bessel's Inequality, Gram Schmidt Process, Application of G-S process to certain Linearly Independent Sequences in  $L_2[0, 2\pi]$ , Orthonormal Basis in Separable Hilbert Spaces.

### Unit-IV

The Conjugate Space, The Adjoint of an Operator on Hilbert Space and its Properties, Self Adjoint Operators, Normal and Unitary Operators, and their Properties, Projections on a Hilbert Space and their Characterization, Reflexivity of Hilbert Space, Riesz Representation Theorem and Finite Spectral Theorem for Normal Operators.

### Text Books:

1. G.F. Simmons, **Introduction to Topology and Modern Analysis**, 4<sup>th</sup> Edition, Tata McGraw Hill Ltd, 2004.
2. E.Kreyszig, **Introductory Functional Analysis with Applications**, John Wiley and Sons (Axia) Pvt Ltd, 2006.
3. J.B. Conway, **A Course in Functional Analysis**, 2nd Edition Springer Verlag, 2006. Springer-Verlag, 1995.

### Reference Books:

1. B.V. Llmaya, **Functional Analysis**, 2nd Edition, New Age International (P) Ltd, 1996.
2. Martin Schechter, **Principles of Functional Analysis**, 2nd Edition, AMS Bookstore, 2002.

3. W. Rudin, **Functional Analysis**, McGraw Hill, Inc., 1991.
4. Kosaku Yoshida, **Functional Analysis**, Springer Verlag 1974.
5. C. Goffman and G. Pedrick, **A First course in Functional Analysis**, Prentice Hall of India, New Delhi.

## Complex Analysis II (MMT-C303)

### Unit-I

Identity Theorem . Maximum Modulus principle and minimum Modulus Principle. Schwarz Lemma and its Generalizations. Hadamard's 3-Circle Theorem, Borel-Carathéodory Theorem. Meromorphic Functions. Argumented principle. Rouché's Theorem and its Applications. Poisson Integral formula for the Circle and Half plane. Poisson-Jensen Formula. Carleman's Theorem.

### Unit-II

Principle of Analytic continuation, Uniqueness of Direct Analytical Continuations and Uniqueness of Analytic Continuation along a Curve. Power series method of Analytic Continuation. Functions with Natural boundaries and related examples. Schwarz reflection principle and its Converse. Functions with Positive real part. Borel's Theorem.

### Unit-III

Space of Analytic Functions, Hurwitz's Theorem, Montel's Theorem, Riemann Mapping Theorem, Weierstrass Factorization Theorem, Gamma Function and its Properties, Riemann Zeta Function, Riemann's Functional Equation. Harmonic functions on a disc, Harnack's Inequality and Harnack's Theorem. Green's Functions.

### Unit-IV

Univalent functions. A Univalent function never vanishes in its Domain. properties of Univalent Functions. Area Theorem. Koebe's Quarter(1/4) Theorem. Bieberbach's Conjecture (statement only). Canonical products. Order of an entire function and some related results. Exponential Convergence and Borel's Theorem. Hadamard's Factorization Theorem. Zeros and Critical points of polynomials. Gauss-Lucas Theorem.

### Text Books:

1. S. PONNUSAMY, **Foundations of Complex Analysis**.
2. Richard A Silverman, **Introductory Complex Analysis**, Prentice Hall, Inc., 1967.
3. J.B. Conway, **Functions of One Complex Variable, II**, Graduate Text in Mathematics, 159, Springer-Verlag, 1995.
4. L.V. Ahlfors, **Complex Analysis**, 3rd Edition, MC Graw Hill, New York, 1979.

### Reference Books:

1. Morris marden, **Geometry of polynomials**.
2. Q.I. Rahaman and G. Schmeisser, **Analytic Theory of Polynomials**.
3. E.C. Titchmarsh, **Theory of Functions**, Oxford University Press.
4. W.H.J. Fuchs, **Topics in Theory of Functions on One Complex Variable**.

5. E.Hille, **Analytic Function Theory**, Vol. 1, Ginn, 1959.
6. R. Nevanlinna, **Analytic Functions**, Springer 1970.
7. M.R. Spiegel, **Theory and Problems of Complex Variables**, Schaums Outline Series, Mc Graw Hill, New York , 1985.
8. R.V. Churchill, J.W. Brown and R.F. Verkey, **Complex Variables and Applications**, 5, Ed. MC- Graw Hill, New York, 1989.
9. E.Hille : **Analytic Function Theory** (2- vol).

## Probabilty and Statistics (MMT-E301)

### Unit-I

The probability set function, its properties, probability density function, the distribution function and its properties. Mathematical Expectations, some special mathematical expectations, Inequalities of Markov, Chebyshev and Jensen.

### Unit-II

Conditional probability, independent events, Bayes theorem, Distribution of two and more random variables, Marginal and conditional distributions, conditional means and variances, Correlation coefficient, stochastic independence and its various criteria.

### Unit-III

Some Special Distributions, Bernoulli, Binomial, Trinomial, Negative Binomial, Poisson, Gamma, Chi-square, Beta, Cauchy, Exponential, Geometric, Normal and Bivariate Normal Distributions.

### Unit-IV

Distribution of Functions of Random Variables, Distribution Function Method, Change of Variables Method, Moment generating function Method, t and F Distributions, Dirichelet Distribution, Distribution of Order Statistics, Distribution of  $X$  , Limiting distributions, Central Limit theorem.

### Text Books:

1. Hogg and Craig : **An Introduction to the Mathematical Statistics**
2. S C Gupta and V K Kapoor: **Mathematical Statistics**,

### Reference Books:

1. Mood and Grayball : **An Introduction to the Mathematical Statistics**

## Measure and Integration (MMT-E302)

### Unit-I

Semiring, algebra and  $\sigma$  algebra of sets, Borel sets, measures on semirings, outer measure associated with a set function and basic properties, measurable sets associated with an outer measure as a  $\sigma$  algebra, construction of the Lebesgue measure on  $R^n$ .

### Unit-II

For  $f \in L_1[a, b]$ ,  $F = \int_a^x f(t) dt$  on  $[a, b]$ . If  $f$  is absolutely continuous on  $(a, b)$  with  $f(x) = 0$  a.e, then  $f = \text{constant}$ . Characterization of an absolutely continuous function as an indefinite Lebesgue integral. Non-Lebesgue integrability of  $f$  where  $f(x) = x^2 \sin(\frac{1}{x^2})$ ,  $f(0) = 0$  on  $[0, 1]$ . Fundamental theorem of calculus for the Lebesgue integral. A brief introduction to  $L_p$  spaces. Holder's and Minkowski's inequalities.

### Unit-III

Improper Riemann integral as a Lebesgue integral, calculation of some improper Riemann integrable functions, space of Lebesgue integrable functions as completion of Riemann integrable functions on  $[a, b]$ , change of variables formula and simple consequences, Riemann Lebesgue lemma.

### Unit-IV

Product measures and iterated integrals, example of non-integrable functions whose iterated integrals exist (and are equal), Fubini theorem, expressing a double integral as an iterated integral, Tonelli-Hobson theorem as a converse to Fubini theorem, differentiation under the integral sign.

### Text Books:

1. C.D. Aliprantis and O. Burkinshaw, **Principles of Real Analysis**
2. H.L. Royden, **Real Analysis**, MacMillan, 1988
3. Walter Rudin, **Principles of Mathematical Analysis**, 3rd Edition. Mc-Graw Hill, 1976.
4. T. M. Apostol, **Mathematical Analysis-Second Edition**, Narosa Publishing House.

### Reference Books:

1. Rana, I.K. : **An Introduction to Measure and Integration**, Narosa
2. Chae, S.B. **Lebesgue Integration** (Springer Verlag).
3. R. Goldberg, **Methods of Real Analysis**, John Wiley and Sons, 1976
4. G. De Barra, **Measure Theory and Integration** Narosa Publishing House, New Delhi

## Discrete Mathematics (MMT-E303)

### Unit-I

Elementary Set theory: The sum rule and the product rule, two-way counting, permutations and combinations, Binomial and multinomial coefficients, Pascal identity, Binomial and multinomial theorems. Arithmetic functions.

### Unit-II

Advanced counting : Types of occupancy problems, distribution of distinguishable and indistinguishable objects into distinguishable and indistinguishable boxes (with condition on distribution) Stirling numbers of second and first kind, Inclusion Exclusion Principle and its application to derangement, Mobius inversion formula.

### Unit-III

Pigeon-hole principle, generalized pigeon-hole principle and its applications, Erdos-Szekers theorem on monotone subsequences. Recurrence relation, solution of homogeneous recurrence relation with constant coefficients. Generating Functions, Solution of Recurrence relations by generating functions.

### Unit-IV

Partially ordered set, Lattices, Distributive and Modular Lattices, complements, Boolean Algebra. Basic definitions, Duality, Basic Theorems. Boolean algebra and Lattices. Sum of products form for Boolean Algebra, logic gates and Circuits

### Text Books:

1. V. Krishnamurthy, **Combinatorics: Theory and applications**, Affiliated East-West
2. Richard A. Brualdi, **Introductory Combinatorics-I**, Pearson
3. A. Tucker, **Applied Combinatorics**, John Wiley and Sons.
4. Norman L. Biggs, **Discrete Mathematics**, Oxford University Press.
5. Kenneth Rosen, **Discrete Mathematics and its applications**, Tata McGraw Hills.
6. Seymour Lipschutz and Marc Lars Lipson, **Discrete Mathematics**, Tata McGraw Hills.

## Number Theory II (MMT-E304)

### Unit-I

Quadratic Residues, Quadratic Reciprocity, The Jacobi symbol. Greatest integer function, Arithmetic functions and their properties. The Mobius inversion formula and Recurrence functions.

### Unit-II

Farey Fractions and Irrational numbers: Farey sequences, Rational approximation, Irrational numbers, The geometry of numbers. Simple Continued Fraction: The Euclidean Algorithm, Uniqueness, Infinite continued fraction, Irrational numbers, Approximation to irrational numbers, Best possible approximation, Periodic continued fraction. (Ch. 1- Ch. 3)

### Unit-III

Primes and Multiplicative Number theory: Elementary prime number estimates, Dirichlet series, Estimates of Arithmetic functions, Primes in Arithmetic Progression.

### Unit-IV

Algebraic Numbers: Polynomials, Algebraic numbers, Algebraic number field, Algebraic integers, Quadratic fields, Units in Quadratic Fields, Primes in Quadratic field, Unique Factorization, Primes in Quadratic Fields having unique factorization property, The equation  $x^3 + y^3 = z^4$ .

### Text Books:

1. Ivan Niven and H.S. Zuckerman, **An Introduction to the Theories of Numbers**, 5th edition, Wiley Eastern, 2000.
2. Leveque, **Topics in Number Theory**, Springer.
3. Boevich and Shaferwich, **Number theory**, I.R. Academic Press.
4. LApostol **Analytic Number theory**, Springer

### Reference Books:

1. E. Landau **Elementary Number theory** AMS Chelsea publishing
2. J. P. Serre **A course in Arithmetic GTM Vol** Springer
3. D.M. Burton, **Elementary Number Theory**, 2nd edition, Universal Book Stall, New Delhi, 1994.
4. Y. Hsiung, **Elementary Theory of Numbers**, World Scientific, 1992; First Indian Reprint, Allied Publishers Limited, 1995.
5. G.H. Hardy and E.M. Wright, **An Introduction to the Theory of Numbers**, 4th edition, Oxford, Clarendon Press, 1960.



6. G.E. Andrews, **Number Theory**, Hindustan Publishing Corporation, New Delhi, 1992.
7. S.G. Telang, **Number Theory**, Tata McGraw Hill Publishing Company Limited, New Delhi, 1996

## **Theory of Semigroups and Non Commutative rings (MMT-E305)**

### **Unit-I**

Basic definitions, group with zero, monogenic semigroups, ordered sets, semilattices and lattices, binary relations, equivalences.

### **Unit-II**

Congruences, free semigroups and monoids, presentation of semigroups, ideals and Rees congruences, lattices of equivalences and congruences.

### **Unit-III**

Wedderburn-Artin Theory, Basic terminology and examples, Jacobson radical theory, Some commutativity problems in rings, Wedderburn Theorem on finite division rings, Jacobson's theorem, Jacobson Herstein Theorem, Kaplansky Theorem.

### **Unit-IV**

Rings of quotients, Goldie rings, Ore domains, Prime Goldie rings, First Goldie theorem, faith Utumi theorem, Semi prime Goldie rings, Prime left ideal rings, nil rings satisfying ascending chain conditions, Nil Goldie rings.

### **Text Books:**

1. J.M. Howie: **Fundamental of Semigroup Theory** (Clarendon Press).
2. R. Keown: **An Introduction to Group Representation Theory** (Academic Press).

### **Reference Books:**

1. T.Y. Lam: **A First Course in Non-commutative rings.**
2. I.N. Herstein: **Topics in Rings Theory**, University of Chicago Press, Chicago.

## Linear Algebra (MMT-C401)

### Unit-I

Algebras, The algebra of Polynomials and Polynomial Ideals. Determinant functions, Permutations and the uniqueness of determinants. Additional properties of determinants, Multilinear functions and The Grassman Ring.

### Unit-II

Characteristic Values, Annihilating Polynomials, Invariant subspaces. Simultaneous Triangulation; Simultaneous Diagonalization, Direct-sum decompositions, Invariant direct sums. The primary Decomposition Theorem, cyclic subspaces and Annihilators, Cyclic decompositions and the rational Form.

### Unit-III

The Jordan Form, Computation of Invariant Factors, Semi-simple Operators. Inner Product Spaces, Linear functionals and Adjoints, Unitary and Normal Operators, Forms on Inner Product spaces. Positive forms and some more results on forms.

### Unit-IV

Spectral Theory, Further properties of Normal operators. Bilinear Forms, Symmetric bilinear forms, Skew-Symmetric Bilinear forms and Groups Preserving Bilinear Forms.

### Text Books:

1. Kenneth Hoffman and Ray Kunze, **Linear Algebra**, 2nd Edition Prentice Hall.

### Reference Books:

1. Gilbert Strang, **Introduction to Linear Algebra**, Wellesley-Cambridge Press.
2. M. Artin , **Algebra** Prentice - Hall of India private Ltd.
3. A.G. Hamilton : **Linear Algebra**, Cambridge University Press (1989).

## Functional Analysis II (MMT-C402)

### Unit-I

Relationship between analytic and geometric forms of Hahn-Banach Theorem, Applications of Hahn-Banach Theorem: Banach limits, Markov-Kakutani theorem for a commuting family of maps, Complemented subspaces of Banach spaces, Complementability of dual of a Banach space in its bidual, uncomplementability of  $C_0$ .

### Unit-II

Dual of Subspace, Quotient space of a normed space. Weak and weak\* topologies on a Banach space, Goldstines theorem, Banach-Alaoglu theorem and its simple consequences. Banachs closed range theorem, injective and surjective bounded linear mappings between Banach spaces.

### Unit-III

$l_\infty$  and  $C[0,1]$  as universal separable Banach spaces,  $l_1$  as a quotient universal separable Banach space, Reflexivity of Banach spaces and weak compactness, Completeness of  $L_p[a,b]$ , Extreme points, Krein-Milman theorem and its simple consequences, Banach Stone Theorem.

### Unit-IV

Duals of  $l_\infty$ ,  $C(X)$  and  $L_p$  spaces. Applications of fundamental theorems to Radon-Nikodym Theorem, Laplace transform. Mazur-Ulam theorem on isometries between real normed spaces, Muntz theorem in  $L_2[a,b]$ .

### Text Books:

1. Ballobas, B; **Linear Analysis** (Camb. Univ.Press).
2. Beauzamy, B; **Indroduction to Banach Spaces and their geometry** (North Holland ).
3. Walter Rudin; **Functional Analysis** (Tata McGrawHill).
4. C.D.Aliprantis and K C Border, **Infinite Dimensional Alaysis** (Springer Verlag, 2006).
5. C. Goffman and G.Pedrick; **A first course in functional Analysis** (Prentice Hall).

## Ordinary and Partial Differential Equations (MMT-C403)

### Unit-I

First order ODE, singular solutions, p-discriminate and c-discriminate, initial value problems of first order ODE, general theory of homogenous and non-homogenous linear ODE, Picards theorem of the existence and uniqueness of solutions to an initial value problem, factorization of operator, variation of parameters, numerical approximation to the solution of differential equations

### Unit-II

Solution in series : Methods of Frobenius (i) Roots of an indicial equation, un-equal and differing by quantity not an integer (ii) Roots of an indicial equation, which are equal (iii) Roots of an indicial equation differing by an integer making coefficient infinite (iv) Roots of an indicial equation differing by an integer making a coefficient indeterminate. Simultaneous equations  $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$  and its solutions by use of multipliers and a second integral found by the help of the first. Total differential equation  $Pdx + Qdy + Rdz = 0$ . Necessary and sufficient condition than an equation may be integrable. Geometric interpretation of the  $Pdx + Qdy + Rdz = 0$ . Partial differential equation : Partial differential equation of the first order, Lagranges linear equation  $P_p + Q_q = R$ , Charpits Method.

### Unit-III

1-D Heat Equation (Physical Derivation) Initial condition and Boundary conditions Separation of variables Complete solution. Curves of constant temperature. Maximum principle for basic heat equation. Equilibrium temperature profile. Error of first term approximation. Well posed problems. Uniqueness of the solution of the heat equation. Variations on the basic Heat equation viz., Dirichlet conditions, Neumann conditions and other boundary conditions. Quasi-steady state, heating/cooling of earth. Linearity, homogeneity and superposition and its application to the solution of the heat equation.

### Unit-IV

1-D wave Equation (Physical Derivation) Initial conditions and Boundary conditions Solution via separation of variables. Normal modes of vibration, amplitude, conservative system and energy. conservation of energy for single and multi-mode wave. D'Alembert solution for wave equation. Characteristics. Simplified solution method for infinite string. waves on an infinite string. Quasi-linear PDE. Traffic flow and its solution via method of characteristics. validity of solution and breakdown. example of light traffic heading into heavier traffic.

### Text Books:

1. A. Coddington and N. Levinson, **Theory of Ordinary Differential Equations**, Tata MC Graw Hill.
2. G.F. Simmons, **Ordinary Differential Equations with Applications and His-**

**torical Notes**, Tata MC Graw Hill, 2005.

3. Guenther, R. B., and J. W. Lee, **Partial Differential Equations of Mathematical Physics and Integral Equations**, New York, NY: Dover Publications, 1996.
4. Myint-U, Tyn, and Lokenath Debnath. **Linear Partial Differential Equations for Scientists and Engineers**. 4th ed. Boston, MA: Birkhauser, 2006.
5. Fritz John, **Partial Differential Equations**, Springer Verlag.

**Reference Books:**

1. Ian Sneddon, **Partial Differential Equations**, McGraw Hill
2. L.C. Evans, **Partial Differential Equations**, GTM, AMS, 1998

## Graph Theory (MMT-E401)

### Unit-I

Introduction of Graphs, Paths and Cycles, Operations on graphs, Bipartite graphs and Konigs Theorem, Euler graphs and Eulers Theorem, Konigsberg bridge problem, Hamiltonian graphs and Diracs Theorem. Degree sequences, Wang-Kleitman Theorem, Havel-Hakimi Theorem, Hakimis Theorem, Erdos- Gallai Theorem, Degree sets.

### Unit-II

Trees and their Properties, Pendant Vertices in trees. Degree sequences in trees. Necessary and Sufficient condition for a sequence to be a degree sequence of a tree. Centers in trees, Binary and rooted trees. Spanning trees, Number of spanning trees of a graph. Cayleys theorem, Fundamental cycles, generation of trees and Elementary tree transformations. Helly property

### Unit-III

Connectivity, Cut-sets and their properties, Seperable Graphs. Blocks and their Properties. Block-Cut Vertex tree.Connectivity Parameters, Vertex connectivity and edge connectivity. Cut of a Graph, Whitney's Theorem, Mengers Theorem (vertex and edge form). Properties of a bonds and cut sets. Fundamental Bonds. Block graphs and Cut-Vertex Graphs.

### Unit-IV

Planar Graphs. Kurtowski's two Graphs. Embedding on a Sphere. Euler's Theorem. Kurotuwski's Theorem. Geometric dual of a planar graph. Whitney's Theorem. Matrices of Graphs. Incidence Matrix  $A(G)$ , Modified Incidence Matrix  $A_f$ , Cycle Matrix  $B(G)$ , Fundamental Cycle Matrix  $B_f$ , Cut-Set Matrix  $C(G)$ , Fundamental Cut-Set Matrix  $C_f$ . Realtion between  $A_f, B_f$  and  $C_f$ . Path Matrix and Adjacency Matrix.

### Text Books:

1. R. Balakrishnan, K. Ranganathan, **A Text Book of Graph Theory**, Springer-Verlag, New York.
2. F. Harary, **Graph Theory**, ddison-Wesley.
3. Narsingh Deo, **Graph Theory with Applications to Engineering and Computer Science**, Prentice Hall.
4. S. Pirzada, **An Introduction to Graph Theory**, Universities Press, Orient Blackswan, 2012.

### Reference Books:

1. B. Bollobas, **Extremal Graph Theory**, Academic Press.
2. JW. T. Tutte, **Graph Theory**, Cambridge University Press.

3. D. B. West, **Introduction to Graph Theory**, PrentiCE HALL.



## Probability and Measure (MMT-E402)

### Unit-I

Measure theory , semi algebra, sigma -algebra, monotone class theorem, Dynkin pi lambda theorem, probability measures, finite measure, sigma finite measure, complete measure, statement of Caratheodory extension theorems. Integration: measurable functions, simple functions etc., monotone convergence theorem, dominated convergence theorem, Fatou's lemma.

### Unit-II

Product spaces, statement of Fubini's theorem. Probability: probability measure on  $\mathbb{R}$ , random variables, distribution function and its properties, independence, Kolmogorov 0-1 law, Borel Cantelli lemma.

### Unit-III

Tchebychev's inequality, Markov's inequality. Various modes of convergence, Weak and Strong law of large numbers. Characteristic functions, statements of the uniqueness, inversion formulae and Levy cramer continuity theorems.

### Unit-IV

Convergence in distribution, convergence with probability 1, Laws of large numbers, CLT for i.i.d . random variables (finite variance case).

### Text Books:

1. Petric, Billingsley, **Probability and Measure**, Wiley.
2. Athreya, Sunder **Measure Theory and Probability Theory**, Springer.
3. Athreya and Lahiri, **Measure and Probability**, CRC Press Inc.

## **Operations Research (MMT-E403)**

### **Unit-I**

Definition and Scope of operations Research (OR). Main Phases of OR Study. Linear Programming problems (LPP), Applications to Industrial Problems and Marketing. Convex Sets and Convex Functions. Simplex Method and Extreme point Theorems. Big M and two Phase methods of Solving an LPP. Revised Simplex method.

### **Unit-II**

Duality in LPP. Formulation of dual problems. Duality Theorems (Weak and Strong Duality). Dual Simplex Method. Primal-Dual Relations. Assignment Problem. Hungarian Method. Transportation Problem (TP) and its Mathematical Formulation. Methods of Solving TP, North-West Corner Rule, Vogel's Method and U-V Method.

### **Unit-III**

Sequencing and Scheduling problems. 2-Machine n-Job and 3-Machine n-Job Problems with identical Machine sequence for all jobs. 2-job n-Machine Problem with different routings. Project Management PERT and CPM. Probability of Completing a project.

### **Unit-IV**

Decision making in the face of Competition. Game Theory. Two person zero sum games. Games with mixed Strategies. Existence of a solution and uniqueness of the value in zero sum games. Equivalence between game Theory and LPP. Simplex Method for Game Problem.

### **Text Books:**

1. H. A. Taha, **Operations Research An Introduction.**, Macmillan Publishing Co. Inc. , N.Y.
2. Kanti Swarup, P.K.Gupta and Man Mohan, **Operations Research**, S. Chand and Sons, New Delhi.
3. J.K. Sharma, **Operations Research-Theory and Application**, Macmillan Pub.

### **Reference Books:**

1. G. Hadley, **Linear Programming**, Narosa Publishing House, 1995.
2. J.K. Sharma, **Operations Research-Problems and Solutions**, Macmillan Pub.
3. F. S. Hillier and G. J. Lieberman, **Introduction to Operations Research**, McGraw Hill International Edition, 1995.

## Commutative Algebra (MMT-E404)

### Unit-I

Zero divisors, Nilpotent elements, units. Prime ideals and maximal ideals, local rings and residue fields. Nil and Jacobson radical. The prime spectrum of a ring and Zarasiki topology. Extension and contraction of ideals. Direct sum and product, Finitely generated modules. Nakayama's Lemma.

### Unit-II

Tensor product of modules, basic properties. Exact sequences, Projective Injective and Flat modules. Restriction and extension of scalars, exactness properties of the tensor and Hom-modules. Algebras, and tensor product of algebras.

### Unit-III

Rings and modules of fractions, Localization of a ring at a prime ideal. Properties of rings and modules of fractions, extended and contracted ideals in rings of fractions. Primary decomposition of ideals 1st and 2nd uniqueness Theorems.

### Unit-IV

Chain conditions, Noetherian and Artinian modules. Noetherian rings, Hilbert basis theorem, Primary decomposition in Noetherian rings. Artin rings and Structure Theorem for Artin rings.

#### Text Book:

1. M.F.Atiyah and I.G.Macdonald, **Introduction to Commutative Algebra** Addison-Weisly publishing company

#### Reference Books:

1. Zariski Oscar and Samuel Pierre, **Commutative Algebra I, II** Springer.
2. H. Matsumura, **Commutative Ring Theory**, Cambridge studies in advanced mathematics.
3. David Eisenbud, **Commutative Algebra with a view towards Algebraic Geometry**, Springer.
4. D.S. Dumit and R.M. Foote, **Abstract Algebra**, 3rd Edition Wiley.

## Category Theory and Infinite Abelian Groups (MMT-E405)

### Unit-I

Definition and examples of a category, small category, sub-category, full sub-category, dual category, monomorphism, epimorphism, bimorphism, section, retraction, isomorphism, balanced category, Initial object, terminal object, zero object connected category, Products and coproducts, Equalizers and coequalizers, Pullbacks and Pushouts, Intersection.

### Unit-II

Factorization of morphisms, kernel and cokernel of a morphism, normal, conormal and bi-normal category, Exact sequence semi additive and additive category, Abelian category, Functors, covariant and contravariant Functors, Natural transformation.

### Unit-III

Commutative diagrams, Direct sums and Direct Products, Direct summands, Free abelian groups, Projective groups, Finitely generated groups.

### Unit-IV

Divisibility, Injective groups, The structure of divisible groups, The divisible hull, Purity, Bounded pure subgroups, Quotient groups module Pure subgroups.

#### Text Books:

1. T.S. Blyth, **Categories**.
2. B. Pareiars: **Categories and Functors**.

#### Reference Books:

1. Mitchell, **Theory of Categories**.
2. Freyd, **Abelian Categories**.
3. Laszle Fuchs, **Infinite Abelian groups**, Vol. 1, Academic Press, New York.