

Department of Mathematics Central University of Kashmir

**Revised Course structure and syllabus for M.A/M. Sc. Mathematics to be operated
w.e.f August 2018**

Semester - I

Course Code	Course Title	Contact Hours per Week (Theory+Tutorials+Presentation)	Credits	Marks	
				CIA	ESE
MMT-C101	Algebra-I	[3+2]	4	40	60
MMT-C102	Real Analysis-I	[3+2]	4	40	60
MMT-C103	Complex Analysis-I	[3+2]	4	40	60
MMT-C104	Topology	[3+2]	4	40	60
MMT-C105	Number Theory-I	[3+2]	4	40	60
SS*	Soft Skills (Elective)	[4]	4	40	60
	Semester Credits/Marks	[19+10]	24	240	360
	Subtotal		24 of 96	600 of 2400	

*To be chosen out of the CBCS courses provided by the University for PG Programmes.

Semester-II

Course Code	Course Title	Contact Hours per Week (Theory+Tutorials+Presentations)	Credits	Marks	
				CIA	ESE
MMT-C201	Linear Algebra	[3+2]	4	40	60
MMT-C202	Real Analysis-II	[3+2]	4	40	60
MMT-C203	Ordinary and Partial Differential Equations	[3+2]	4	40	60
MMT-C204	Differential Geometry	[3+2]	4	40	60
MMT-C205	Discrete Mathematics	[3+2]	4	40	60
SO*	Social Orientation Elective	[4]	4	40	60
	Semester Credits/Marks	[19+10]	24	240	360
	Subtotal		48 of 96	1200 of 2400	

*To be chosen out of the CBCS courses provided by the University for PG Programmes.

Semester-III

Course Code	Course Title	Contact Hours per Week (Theory+Tutorials+Presentations)	Credits	Marks	
				CIA	ESE
MMT-C301	Functional Analysis-I	[3+2]	4	40	60
MMT-C302	Complex Analysis-II	[3+2]	4	40	60
MMT-C303	Algebra-II	[3+2]	4	40	60
ELECTIVE COURSES (Any two of the following)					
Course Code	Course Title	Contact Hours per Week (Theory+Tutorials+Presentations)	Credits	Marks	
				CIA	ESE
MMT-E304	Probability Theory	[3+2]	4	40	60
MMT-E305	Abstract Measure Theory	[3+2]	4	40	60
MMT-E306	Operation Research	[3+2]	4	40	60
MMT-E307	Topics in Graph Theory	[3+2]	4	40	60
MMT-E308	Topics in Ring Theory	[3+2]	4	40	60
MMT-E309	Calculus of Variation and Integral Equations	[3+2]	4	40	60
MMT-E310	Fourier Analysis	[3+2]	4	40	60
OGE*	Open Generic Elective	4	4	40	60
	Semester Credits/Marks	[18+12]	24	240	360
	Subtotal		72 of 96	1800 of 2400	

*To be chosen out of the CBCS courses provided by the University for PG Programmes.

Semester-IV

Course Code	Course Title	Contact Hours per Week (Theory+Tutorials+Presentations)	Credits	Marks	
				Viva	Dissertation
MMT-C401	Project*	[5+3]	8	80	120
ELECTIVE COURSES (<i>Any FOUR of the following</i>)					
Course Code	Course Title	Contact Hours per Week (Theory+Tutorials+Presentations)	Credits	Marks	
				CIA	ESE
MMT-E402	Functional Analysis-II	[3+2]	4	40	60
MMT-E403	Commutative Algebra	[3+2]	4	40	60
MMT-E404	Number Theory II	[3+2]	4	40	60
MMT-E405	Theory of Semigroups	[3+2]	4	40	60
MMT-E406	Probability and Measure	[3+2]	4	40	60
MMT-E407	Wavelet Analysis	[3+2]	4	40	60
MMT-E408	Topics in the Analytic Theory of Polynomials	[3+2]	4	40	60
MMT-E409	Banach Algebras	[3+2]	4	40	60
MMT-E410	Coding Theory	[3+2]	4	40	60
MMT-E411	Complex Dynamics	[3+2]	4	40	60
MMT-E412	Fluid Dynamics	[3+2]	4	40	60
MMT-E413	Category Theory	[3+2]	4	40	60
	Semester Credits/Marks	[17+11]	24	240	360
				600	
	Subtotal		96 of 96	600 of 2400	

*Students must write a project on any one of the topic allotted by the concerned supervisor and submit it in the form of dissertation. The same is to be evaluated by an expert and then it has to be defended by the student in a viva before the panel of experts.

Total marks for the programme =2400

Total credits of the programme= 96

C- Stands for **CORE** and E-Stands for **ELECTIVE**

MMT-C101 Algebra-I

Unit-I

Review of groups and subgroups with special reference to permutation groups, groups of symmetries and Dihedral groups. Cyclic groups and cyclic subgroups. Subgroups generated by a subset of a group. Centralizer, normalizer, normal subgroups and quotient groups. Group homomorphisms and Isomorphism Theorems. Lagrange's and Cauchy's Theorems.

Unit-II

Group actions and permutation representations. Stabilizers, kernels and orbits of group actions, cyclic decompositions. Groups acting on themselves by left Multiplication-Caley's Theorem. Groups acting on themselves by Conjugation-Class equation. Automorphisms and Inner Automorphisms. Sylow's Theorem with examples and applications.

Unit-III

Review of rings and subrings with examples on integers modulo n , quadratic integers rings, polynomial rings, matrix rings and group rings. Integral domains, fields and related results. Ideals and quotient rings. Ring homomorphisms and Isomorphism Theorems. Properties of ideals, prime and maximal ideals. Rings of fractions and embedding Theorems.

Unit-IV

Euclidean Domains, Principal ideal Domains and Unique Factorization Domains with related results and examples. Polynomial rings over fields, Polynomial rings that are unique factorization domains. Irreducibility criteria for polynomials.

Text Book:

1. *Abstract Algebra*, D.S. Dumit and R.M. Foote, Wiley 3rd edition.

Suggested Readings:

1. *Topics in Algebra*, I.N. Herstein, Wiley.
2. *Basic Abstract Algebra*, Bhattacharyya and Nagpaul, Cambridge University press.
3. *Contemporary Abstract Algebra*, Joseph Gallian, Narosa Publishing House.
4. *University Algebra*, N S Gopal Krishnan, New age Internationals.

MMT-C102: Real Analysis-I

Unit-I

Inequalities: Arithmetic Mean- Geometric mean inequality, Cauchy-Schwarz inequality, Chebyshev's inequality, Holder's and Minkowski inequalities, Convex and concave functions, Jensen's inequality, Bernoulli's inequality, some applications involving inequalities

Unit-II

Improper Integrals: Integration of un-bounded functions with finite limit of integration, Comparison of tests for convergence of improper integrals, Cauchy's test for convergence. Absolute convergence, Infinite range of integration of bounded functions, convergence of integrals of unbounded functions with infinite limits of integration, integrated as a product of functions, Abel's and Dirichlet's tests of convergence.

Unit-III

Uniform convergence of sequences and series of functions. Pointwise convergence, uniform convergence on an interval, Cauchy's criterion for uniform convergence, Mn-test for uniform convergence of sequences, Weierstrass's M-Test, Abel's and Dirichlet's tests for uniform convergence of series. Uniform convergence and continuity, uniform convergence and integration and uniform convergence and differentiation, Weierstrass's Approximation Theorem.

Unit-IV

Convergence of Fourier series: Dirichlet's Criterion. Fejer's theorem on $(C,1)$ -convergence of Fourier series of a continuous function on $C(-\pi, \pi)$, Partial derivative, Limits, Continuity and Differentiability of functions from \mathbb{R}^n to \mathbb{R} . Chain rule. Sufficient conditions for differentiability and for the equality of mixed partials, Taylor's theorem for functions from \mathbb{R}^2 and \mathbb{R} . Implicit and Inverse function theorems.

Suggested Texts:

1. Walter Rudin, "Principles of Mathematical Analysis", 3rd Edition. Mc-Graw Hill, 1976.
2. R. Goldberg, "Methods of Real Analysis", John Wiley & Sons, 1976
3. T.M. Apostol, "Mathematical Analysis", Narosa, 2004.

Reference Books:

1. S. Lang, "Real Analysis", Addison, Wesley, 1969
2. S.C. Malik, "Mathematical Analysis" Wiley Eastern Limited Urvashi, Press, 1983, Meerut.
3. H.L. Royden, "Real Analysis", MacMillan, 1988
4. G.B. Folland, "Real Analysis", CRC Press.

MMT-C103: Complex Analysis-I

UNIT-I:

Functions of a complex variable, Limits, Continuity, Differentiability, Cauchy-Riemann Equations and their applications, Analytic function, Harmonic function, The functions like $e^z, \sin z, \cos z$ and the complex logarithm. Contour integral, Cauchy's theorem, Cauchy-Goursat's theorem, Cauchy's integral formula, Higher order derivatives, Morera's theorem, Cauchy's inequality, Liouville's theorem and its applications, Winding numbers-index of a point with respect to a closed curve.

UNIT-II:

Power Series, Radius of convergence of a power series, Cauchy's-Hadamard formula for finding radius of convergence, Taylor theorem, Taylor's series, Expansion of analytic functions in a power series, Laurent's series, Singular Points, Isolated singularities, Poles and essential singular points, Behavior of functions at infinity, Casorati – Weirestrass's Theorem.

UNIT-III:

Bilinear transformations- Properties and Classification, Fixed Points, Cross ratios, Inverse points and Critical points. Conformal mapping, Mappings of: Upper half plan on to unit disc, Unit disc onto unit disc, left half plan on to unit disc, Circle onto circle. The transformations: $w = \sqrt{z}$, and $w = z^2$, $w = \frac{1}{2}(z + 1/z)$.

UNIT-IV:

Residues: Cauchy's residue theorem and its applications, Calculation of residues, Evaluation of definite integrals by the method of residues, Perceval's identity. Branches of many valued functions with reference to $Arg(z), \log(z), z^a$. Infinite products, Convergence and divergence of infinite products.

Recommended Books:

1. L. Ahlfors, Complex analysis
2. Richard Silverman, Complex Analysis
3. S. Ponnusamy, Foundations of Complex analysis
4. J.B. Conway, Functions of a complex variable-I

References:

1. Z. Nihari, Conformal mapping.
2. E.C. Titchmarsh, Theory of functions

MMT-C104-TOPOLOGY

UNIT-I

Review of countable and uncountable sets, Schroeder-Bernstein theorem, axiom of choice and its various equivalent forms, definition and examples of metric spaces, open and closed sets, completeness in metric spaces, Baire's category theorem, and applications to the (i) non-existence of a function which is continuous precisely at irrationals (ii) impossibility of approximating the characteristic of rationals on $[0, 1]$ by a sequence of continuous functions.

UNIT-II

Completion of a metric space, Cantor's intersection theorem with examples to demonstrate that each of the conditions in the theorem is essential, uniformly continuous mappings with examples and counter examples, extending uniformity continuous maps, Banach's contraction principle with applications to the inverse function Theorem in \mathbb{R} .

UNIT-III

Topological spaces; definition and examples, elementary properties, Kuratowski's axioms, continuous mappings and their characterizations, pasting lemma, convergence of nets and continuity in terms of nets, bases and sub bases for a topology, lower limit topology, concepts of first countability, second countability, separability and their relationships, counter examples and behaviour under subspaces, weak topology generated by a family of mappings, product topology

UNIT-IV

Compactness and its various characterizations, Heine-Borel theorem, compactness, sequential compactness and total boundedness in metric spaces, Lebesgue's covering lemma, continuous maps on a compact space, Tychonoff's theorem, separation axioms T_i ($i=1,2,3,1\setminus 2,4$) and their permanence properties, connectedness, local connectedness, their relationship and basic properties, connected sets in \mathbb{R} , Urysohn's lemma, Urysohn's metrization theorem, Tietze's extension theorem, one point compactification.

Suggested Texts:

1. G.F. Simmons, Introduction to Topology and Modern Analysis, Tata McGraw Hill.
2. J. Munkres, Topology, Pearson India.

References:

1. K.D. Joshi, Introduction to General Topology.
2. J. L. Kelley, General Topology.
3. Murdeshwar, General Topology.
4. S.T. Hu, Introduction to General Topology.
5. Dugundji J, Topology Prentice Hall of India.
6. Willard, "General Topology" Addison-Wesley
7. I.M. Singer & J.A. Thorpe, "Lecture Notes on Elementary Topology and Geometry".

MMT-C105- NUMBER THEORY-I

Unit-I

Divisibility, Division algorithm, greatest common divisor and its properties, Least Common Multiple, prime numbers, Euclid's first theorem, factorisation of primes, fundamental theorem of arithmetic, linear Diophantine equations, sequence of primes, Euclid's second theorem, infinitude of primes of the form $4n+3$ and of the form $6n+5$, No polynomial with integral coefficients which is not a constant can represent primes for all integral values; ϕ Fermat numbers and properties, Mersenne numbers, arbitrary large gaps in the sequence of primes.

Unit-II

Congruences, complete residue system (CRS), reduced residue system (RRS) and their properties, Fermat Little Theorem and Euler's theorems with applications, Euler's ϕ -function, $\phi(mn) = \phi(m)\phi(n)$ where $(m, n) = 1$, Wilson's theorem and its application to the solution the congruence of $x^2 \equiv -1 \pmod{p}$. Solutions of linear congruence's, Chinese Remainder theorem, congruences of higher degree.

Unit-III

$F(x) \equiv 0 \pmod{m}$, where $F(x)$ is a polynomial, congruences with prime power, congruences with prime modulus and related results, Lagrange's theorem, The polynomial congruence $F(x) \equiv 0 \pmod{m}$, of degree n having at most n solutions, The polynomial congruence $F(x) \equiv 0 \pmod{m}$, of degree n having exactly n solutions, factor theorem and its generalization, polynomial congruences $F(x_1, x_2, \dots, x_n) \equiv 0 \pmod{p}$ in several variables, equivalence of polynomials, Chevalley's theorem, Warning's theorem

Unit-IV

Quadratic forms over a field of characteristic $\neq 2$, equivalence of quadratic forms, Witt's theorem, representation of Field Elements, Hermite's theorem on the minima of a positive definite quadratic form and its application to the sum of two squares. Integers belonging to a given exponent mod p and related results; Converse of Fermat's Little Theorem; If $d|(p-1)$, the congruence $x^d \equiv 1 \pmod{p}$ has exactly d -solutions. If any integer belongs to $t \pmod{p}$, then exactly $\phi(t)$, incongruent numbers belong to $t \pmod{p}$, primitive roots, existence of $\phi(p-1)$ primitive roots for a given odd prime p , the function $\lambda(m)$ and its properties, primitive roots of m . The number of primitive roots of $1, 2, 4, p^\alpha$ and $2p^\alpha$.

Text Books:

1. W. J. Leveque, Topics in Number Theory, Vol. I and II Addition Wesley Publishing Company, INC.
2. I. Niven and H.S Zuckerman, An introduction of the Theory of Numbers.
3. Elementary Number Theory, E. Landau, Chelsea Publishing Co., NY.

Suggested Readings:

1. Analytic Number Theory by T.M. Apostol (Springer Verlag).
2. An Introduction to the theory of Numbers by G.H. Hardy and Wright Oxford Univ Press.

MMT-C201: Linear Algebra

Unit-I

Review of Linear transformations and their properties. The algebra of linear transformations. Representation of transformations by matrices, change of basis and equivalence of matrices. Similarity of matrices and operators, Linear functionals and dual space. Transpose of a linear transformation. Characteristic values, Characteristic and minimal polynomial of a linear operator and diagonalizable linear operators. Invariant subspaces and reducing pairs.

Unit-II

Inner product spaces with examples and properties, Adjoint of operators on inner product spaces. Unitary and Normal operators on inner product spaces with related results. Forms on inner product spaces :(sesqui-linear, bilinear and quadratic).

Unit-III

Modules: basic properties, submodules, spanning sets, Linear independence, torsion elements and annihilators. Free modules, module homomorphisms and quotient modules, The Correspondence and Isomorphism Theorems. Direct sums, and extension of Isomorphisms. Rank of a free module, Noetherian modules. Modules over PIDs, decomposition of cyclic modules, Torsion free and free modules.

Unit-IV

The primary decomposition Theorem and the cyclic Decomposition Theorem of a primary module (proofs may be skipped). Elementary divisors and invariant factor decomposition, module associated with a linear operator, cyclic submodules and cyclic subspaces, indecomposable modules and companion matrices. Rational canonical form (elementary divisor and invariant factor version). Spectral mapping Theorem, Geometric and algebraic multiplicities of eigen values. The Jordan canonical form and Shur's Theorem (Only real case).

Text Books:

1. Linear Algebra, K Hoffman and R Kunze, Pearson India.
2. Advanced Linear Algebra, Steven Roman, Springer Graduate Texts.
3. Abstract Algebra, D.S. Dumit and R.M. Foote, Wiley 3rd edition.

MMT-C202: Real Analysis-II

Unit-I

Partial derivative, directional derivative, total derivative continuity and their relationships of function $f: \mathbb{R}^n \rightarrow \mathbb{R}$. Matrix of a linear function and Jacobian of a differentiable function at a point, chain rule, mean value theorem for differentiable functions. Taylor's theorem for functions from \mathbb{R}^n to \mathbb{R} . Implicit and Inverse function theorems in \mathbb{R}^n . Extremum problems for functions on \mathbb{R}^n . Lagrange's multipliers, Multiple Riemann Integral and change of variables formula for multiple Riemann integrals.

UNIT - II

Semi rings, algebras and σ -algebras of sets, Borel σ -algebra of a topological space, Measures on semirings and examples, Outer measure and measurable sets, Collection of measurable sets as a σ -algebra Σ , σ -additivity of an outer measure on Σ , Caratheodory extension of an outer measure, Length function of an interval as an outer measure on the semiring of finite intervals, Lebesgue measure and existence of Lebesgue non-measurable sets. Lebesgue measure on \mathbb{R}^n .

UNIT-III

Measurable functions and their characterization. Algebra of measurable functions, Stienhauss's Theorem on sets of positive measure, Ostrovisk's theorem on measurable solution of $f(x+y) = f(x) + f(y)$, $x, y \in \mathbb{R}$. Convergence a.e., convergence in measure and almost uniform convergence, their relationship on sets of finite measure, Egoroff's Theorem.

UNIT-IV

Simple and step functions, Integral of a step function, Linearity, monotonicity and order continuity of the integral of step function. Upper function, Integral of upper function, Lebesgue integrable functions: Lebesgue integral of a function in [terType equation here.ms](#) of upper functions, Levi's Theorem, Fatous Lemma, Monotone Convergence Theorem, The Lebesgue dominated Convergence Theorem. The Riemann integral as a Lebesgue integral, Fundamental Theorem of calculus.

Text Books:

1. H.L. Royden, Real Analysis, MacMillan.
2. Walter Rudin, Principles of Mathematical Analysis, 3rd Edition. Mc-Graw Hill.
3. T. M. Apostol, "Mathematical Analysis-Second Edition, Narosa Publishing House.
4. D.V. Widder, Advanced Calculus. 2/e, Prentice Hall of India, New Delhi

Reference Books:

5. R. Goldberg, Methods of Real Analysis, John Wiley and Sons.
6. T.M. Apostol, Mathematical Analysis, Narosa.
7. G. De Barra, Measure Theory and Integration Narosa Publishing House.

MMT-C 203: ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

Unit I

Solution in Series: (i) Roots of an Indicial equation, un-equal and differing by a quantity not an integer. (ii) Roots of an Indicial equation, which are equal. (iii) Roots of an Indicial equation differing by an integer making a coefficient infinite. (iv) Roots of an Indicial equation differing by an integer making a coefficient indeterminate. Simultaneous equation $dx/P = dy/Q = dz/R$ and its solutions by use of multipliers and a second integral found by the help of first. Total differential equations $Pdx + Qdy + Rdz = 0$. Necessary and sufficient condition for integrability of an equation. Geometric interpretation of the $Pdx + Qdy + Rdz = 0$.

Unit II

Existence of Solutions, Initial value problem, Ascoli- lemma, Lipschitz condition and Gronwall inequality, Picard's Theorem on the existence and uniqueness of solutions to an initial value problem, Existence and uniqueness of solutions with examples, Method of successive approximation, Continuation of solutions, System of Differential equations, Dependence of solutions on initial conditions and parameters.

UNIT III

Partial Differential Equations of first order PDEs, origins of first order PDEs, Cauchy Problem for first order equations, Linear equations of the first order, Nonlinear PDEs of the first order, Lagrange and Charpit's methods for solving first order PDEs Classification of 2nd order PDE, General solution of higher order PDEs with constant coefficients, Method of separation of variables for three basic equations: Laplace, Heat and Wave equations.

Unit IV

Wave equation, space-like surface and time-like direction for the 3-D wave equation, D'Alembert's solution, the initial value problem in three space, Poisson's method of spherical averages, Hadamard's method of descent, Duhamel's Principle, the inhomogeneous wave equation. Mean value property and maximum principles for elliptic problems and maximum principle for heat equation. Green's function for half space and disc for the Laplace operator.

Recommended Books:

1. G. F. Simmons: Differential Equations, Tata McGraw Hill.
2. E. A. Coddington and N. Levinson: Theory of Ordinary Differential Equations, Dover.
3. D. Somasundaram, Ordinary Differential Equations, Narosa Publishers.
1. Partial Differential Equations by Fritz John, Springer Verlag.
4. Partial Differential Equations by Amarnath, Alpha Science.

MMT-C 204: DIFFERENTIAL GEOMETRY

UNIT I

Curves: differentiable curves, regular point, parameterization of curves, arclength, arc-length is independent of parameterization, unit speed curves, plane curves, curvature of plane curves, osculating circle, centre of curvature. computation of curvature of plane curves, directed curvature, examples, straight line, circle, ellipse, tractrix, evolutes and involutes, space curves, tangent vector, unit normal vector and unit binormal vector to a space curve, curvature and torsion of a space curve, the Frenet-Serret theorem, first fundamental theorem of space curves, intrinsic equation of a curve, computation of curvature and torsion, characterization of helices and curves on sphere in terms of their curvature and torsion, evolutes and involutes of space curves.

UNIT II

Surfaces: regular surfaces with examples, coordinate charts or curvilinear coordinates, change of coordinates, tangent plane at a regular point, normal to the surface, orientable surface, differentiable mapping between regular surfaces and their differential, fundamental form or a metric of a surface, line element, invariance of a line element under change of coordinates, angle between two curves, condition of orthogonality of coordinate curves, area of bounded region, invariance of area under change of coordinates.

UNIT III

Curvature of a Surface: normal curvature, Euler's work on principal curvature, qualitative behaviour of a surface near a point with prescribed principal curvatures, the Gauss map and its differential, the differential of Gauss is self adjoint, second fundamental form, normal curvature in terms of second fundamental form. Meunier theorem, Gaussian curvature, Weingarten equation, Gaussian curvature $K(p) = (eg-f^2)/EG-F^2$, surface of revolution, surfaces with constant positive or negative Gaussian curvature, Gaussian curvature in terms of area, line of curvature, Rodrigues's formula for line of curvature, equivalence of surfaces, isometry between surfaces, local isometry, and characterization of local isometry.

UNIT IV

Christoffel symbols, expressing Christoffel symbols in terms of metric coefficients and their derivative, Theorem an egerium (Gaussian curvature is intrinsic), isometric surfaces have same Gaussian curvatures at corresponding points, Gauss equations and Manardi Codazzi equations for surfaces, fundamental theorem for regular surface. (Statement only).

Geodesics: geodesic curvature, geodesic curvature is intrinsic, equations of geodesic, geodesic on sphere and pseudo sphere, geodesic as distance minimizing curves. Gauss-Bonnet theorem (statement only), geodesic triangle on sphere, implication of Gauss-Bonnet theorem for geodesic triangle.

Recommended Books:

1. John Mc Cleary, Geometry from a differentiable Viewpoint. (Cambridge Univ. Press).
2. Andrew Presley, Elementary Differential Geometry (Springer Verlag, UTM).
3. Barret O' Neil, Elementary Differential Geometry, Cambridge Univ. Press.

Suggested Readings:

1. W. Klingenberg, A course in Differential Geometry (Spring Verlag).
2. C. E. Weatherburn, Differential Geometry of Three dimensions.

MMT- C205: Discrete Mathematics

UNIT –I

Partially ordered sets, ordered preserving mappings and isomorphism of ordered sets. Lattices and semi lattices with examples and related results. Lattice Homomorphism and complete lattices. Modular and Distributed lattices with examples and related results.

UNIT-II

Chain Conditions, Schreier's theorem. Decomposition theory for lattices with chain Condition, Complemented modular lattices, Distributive lattices. Boolean algebra and Boolean rings

UNIT-III

Graphs: Basic Graph Terminology, Walks, Paths, Cycles, connectedness, isomorphism, Eulerian and Hamiltonian graphs, Necessary and sufficient conditions for Eulerian and Hamiltonian graphs, Dirac's Theorem, Konigsberg Bridge problem and travelling salesman Problem, Tress and characterizations. Rooted and Binary Trees, Enumeration of spanning trees, Kruskal's algorithm, Examples, trees, minimum spanning trees, Cayley's Theorem.

UNIT IV

Degree Sequence, Necessary and sufficient condition for degree sequences, Havel- Hakimi Theorem, Wang and Kleitman theorem, Erdos-Gallai Theorem, Colorings of graphs, Edge colouring, vertex colouring, Total chromatic number, Konigs Theorem, Vizing-Gupta Theorem, The Five color theorem and the statement of the Four-color Theorem, Distance in graph and Metric, Examples, Prove that the distance between the vertices of a connected graph is Metric.

Suggested Texts:

- 1 R. Balakrishnan, Ranganathan, A Text Book of Graph Theory, Springer- Verlag.
- 2 B. Bollobas, Extremal Graph Theory, Academic Press.
- 3 Narsingh Deo, Graph Theory with Applications to Engineering and Computer Science, Prentice Hall.
- 4 T.S. Blyth, Lattices and ordered Algebraic Structures, Springer
- 5 G. Birkhoff, Lattice Theory, AMS, Colloquium Publications, XXV, 1940, third edition 1967.
- 6 J.C. Abbott, sets, lattices and Boolean algebras, Allyn and Bacon, Boston, 1964.

MMT-C301: Functional Analysis-I

Unit I

Banach Spaces: Definition and examples, subspaces, quotient spaces, Continuous Linear Operators and their Characterization, Completeness of the space $L(X, Y)$ of bounded linear operators (and its converse), incompleteness of $C[a, b]$, under the integral norm, Finite dimensional Banach spaces, Equivalence of norms on finite dimensional space and its consequences, Dual of a normed linear space, Hahn Banach theorem (extension form) and its applications, Complemented subspaces, Duals of C_0 , l_p ($p \geq 1$), $C[a, b]$.

Unit II

Uniform boundedness Principle and weak boundedness, Dimension of an n -dimensional Banach space, Conjugate of a continuous linear operator and its properties, Banach-Stienhauss's theorem, open Mapping and closed graph theorems, counterexamples to Banach-Stienhauss's, open mapping theorem and closed graph theorems for incomplete domain and range spaces, separable Banach spaces and the separability of some concrete Banach spaces (C_0 , $C[0,1]$, l_p , $p \geq 1$), Reflexive Banach Spaces, closed subspace and the dual of a reflexive Banach space, Examples of reflexive and non-reflexive Banach spaces.

Unit III

Hilbert spaces: Definition and examples, Cauchy's Schwartz inequality, Parallelogram law, orthonormal (o.n) systems, Bessel's inequality and Parseval's Identity for complete orthonormal systems, Riesz-Fischer theorem, Gram Schmidt process, o.n basis in separable Hilbert spaces. Fourier Series: Fourier series with respect to an o.n. base in Hilbert space, Examples of special o.n. bases in $L_2[0, 2\pi]$.

UNIT-IV

Projection theorem, Riesz Representation theorem. Counterexample to the Projection theorem and Riesz Representation theorem for incomplete spaces. Hilbert property of the dual of a Hilbert space and counter examples of incomplete inner product spaces, Reflexivity of Hilbert space, Adjoint of a Hilbert space operator, weak convergence and by Bolzano-Weirestrass's property in Hilbert Spaces. Normal and Unitary operators, Finite dimensional spectral theorem for normal operators.

Recommended books:

- 1.B.V. Limaya: Functional Analysis.
- 2.C. Goffman G. Pederick: A First Course in Functional Analysis.
- 3.L.A. Lusternick & V.J. Sobolov: Elements of Functional Analysis
- 4.J.B. Conway: A Course in Functional Analysis

MMTC302: Complex Analysis- II

Unit-I

Maximum Modulus Principle, Schwarz Lemma and its generalization, Meromorphic function, Argument Principle, Rouché's theorem with application, Inverse function Theorem, Poisson integral formula for a circle and half plane, Poisson Jensen formula, Carleman's theorem, Hadamard three-circle theorem and the theorem of Borel and Caratheodory.

Unit-II

Principle of analytic continuation, uniqueness of direct analytical continuations and uniqueness of analytic continuation along a curve. Power series method of analytic continuation, Functions with natural boundaries and related examples. Shewartz reflection principle, functions with positive real part.

Unit-III

Space of analytic functions, Hurwitz's theorem, Montel's theorem, Riemann Mapping theorem, Weistrass factorization theorem, Gamma function and its properties. Riemann Zeta function, Reimann's functional equation. Harmonic functions on a disc, Harnack's inequality and theorem, Dirichlet's problem, Green's functions.

Unit IV

Canonical products, order of an entire functions, Exponential convergence, Borel theorem, Hadamard's factorization theorem, The Range of analytic functions, Bloch's Theorem, Schottky's Theorem, The Little Picard's Theorem, Landau's Theorem, Great Picard Theorem (statement and applications only), Univalent function. Bieberbach's conjecture (statement only) and the $1/4$ – theorem.

Text Books:

1. *Complex Analysis*, L. Ahlfors, Springer.
2. *Theory of Functions*, E.C. Titchmarsh Oxford University Press

References:

1. *Functions of a complex variable –I*, J.B. Conway, Springer.
2. *Complex Analysis*, Richard Silverman, Dover publications.
3. *Theory of Functions of a Complex variable*, A. I. Markushevish,

MMT- C303: Algebra-II

Unit-I

Detailed look at the symmetric group S_n . Conjugacy in S_n and simplicity of A_n for $n \geq 5$. p-groups, solvable groups and nilpotent groups. Composition series for groups and Jordan Holder Theorem. Free groups.

Unit-II

Direct products of groups. Fundamental Theorem for finitely generated abelian groups. Invariant factors and Elementary divisors of a group. Isomorphism types of groups of small order. semidirect products of groups. Classifications of groups of order n for certain values of n.

Unit-III

Characteristic of a field, prime subfields and field extensions. Irreducible polynomial and existence of extension field containing their roots. Algebraic extensions and classical straight edge and compass constructions.

Unit-IV

Splitting fields and algebraic closures. Separable and inseparable extensions. Cyclotomic polynomials and extensions. Galois groups and Galois extensions. Fundamental Theorem of Galois Theory. Finite fields and related results.

Text Book:

1. *Abstract Algebra*, D.S. Dumit and R.M. Foote, Wiley 3rd edition.

Suggested Readings:

5. *Topics in Algebra*, I.N. Herstein, Wiley.
6. *Basic Abstract Algebra*, Bhattacharyya and Nagpaul, Cambridge University press.
7. *University Algebra*, N.S. Gopal Krishnan, New age Internationals.
8. *Contemporary Abstract Algebra*, Joseph Gillian, Narosa.

MMT-E304 Probability Theory

UNIT-I

Random Variables. Discrete and Continuous random variables, Expectation of a Random Variable. Some special Expectations. Moment Generating Function. Markov's Inequality and Chebyshev's Inequality. The Binomial and Related Distributions, the Poisson Distribution.

UNIT-II

The Gamma, Chi-Square and Uniform Distribution of functions of random variables with applications. Distribution of two random variables. Bivariate Random variables, Conditional distribution and Expectations, The Correlation coefficient. Stochastic independence.

UNIT-III

Normal Distribution and Multivariate Normal Distribution with applications. Transformations of random variables, distribution of sum of independent random variables, Distribution of function of random variables. The t-Distribution and F-Distribution with applications. Student's Theorem.

UNIT-IV

Limiting Distribution, Convergence in probability, Convergence in probability is closed under limiting distribution. Convergence in Distribution, Poisson distribution is the limiting case of Binomial distribution. Central Limiting Theorem with applications.

Recommended books:

1. Hogg and Craig, Mathematical statistics, Pearson.
2. P.G. Hoel and S.C. Port, Universal Book Stall, New Delhi
3. William Feller, An Introduction to Probability Theory and Applications, Vol. I, Wiley eastern, New Delhi.

MMT-E305: ABSTRACT MEASURE THEORY

UNIT-I

Absolute continuity and bounded variation, their relationships and counter examples, Indefinite integral of a L^1 -integrable functions and its absolute continuity. Necessary and sufficient condition for bounded variation. Vitali's covering lemma and a.e. differentiability of a monotone function f and $\int_a^b f' \leq f(b) - f(a)$.

Unit-II

For $f \in L^1[a, b]$, $F' = f$ a.e. on $[a, b]$. If f is continuous on (a, b) with $f(x) = 0$ a.e, then $f = \text{constant}$. Characterization of a continuous function as an indefinite Lebesgue integral. Non-Lebesgue integrability of f where $f(x) = x^2 \sin(1/x^2)$, $f(0) = 0$ on $[0, 1]$. Fundamental theorem of calculus for the Lebesgue integral. A brief introduction to L_p spaces. Holder's and Minkowski's inequalities.

Unit-III

Improper Riemann integral as a Lebesgue integral, differentiation under the integral sign, calculation of some improper Riemann integrable functions, space of Lebesgue integrable functions as completion of Riemann integrable functions on $[a, b]$, Riemann Lebesgue lemma.

Unit-IV

Product measures and iterated integrals, example of non-integrable functions whose iterated integrals exist (and are equal), Fubini Theorem, expressing a double integral as an iterated integral, Tonelli-Hobson theorem as a converse to Fubini Theorem, change of variables formula and simple consequences.

Recommended Books:

- 1.C.D. Aliprantis and O. Burkinshaw, Principles of Real Analysis, Academic Press.
- 2.Goldberg, R.: Methods of Real Analysis
- 3.T.M. Apostol: Mathematical Analysis, Narosa.

Suggested Readings:

- 1.Royden, L: Real Analysis (PHI)
- 2.Chae, S.B. Lebesgue Integration (Springer Verlag).
- 3.Rudin, W. Principles of Mathematical Analysis (McGraw Hill).
- 4.Barra, De. G: Measure theory and Integration.

MMT-E306: Operations Research

Unit I:

Definition of Operation Research. Features of OR modelling in OR and classification. Methodology of operation research. Opportunities and short comings of operation research, applications of operation research. Applications in business, technology, warfare, education and career, counselling, Operations Research Models.

Unit II:

Linear Programming Problem (LPP): General LPP models, Formulation of LPP models, Graphical solution of LPP. Convex sets, Convex hull, Convex and concave functions, Extreme point theorems and development of Simplex Method, Artificial Variable Technique-Big M-Method and Two-Phase Method.

Unit III:

Formulation of Transportation Problem, Finding of Basic Feasible solution (BFS) using North-West Corner Rule, Matrix Minima and Vogel's Approximation Method, Testing for optimality of the basic feasible solution– MODI and Stepping Stone Methods, Assignment Problem and its formulation, finding an optimal assignment using Hungarian Method.

Unit IV:

Project scheduling: Network representation of a Project, Rules for construction of a Network. Use of Dummy activity. The Critical Path Method (CPM) for constructing the time schedule for the project. Various Float times of activities. Programme Evaluation and Review Technique (PERT). Probability considerations in PERT. Probability of meeting the scheduled time. PERT Calculation, Distinctions between CPM and PERT.

Text Books:

1. *Linear Programming*, Problem, Gauss S.I, John Wiley
2. *Linear Programming*, Hadley, G, Narosa Publishing House.

References:

1. *Operations Research-Principles and Practice*, [Ravindran](#) , A. [Phillips](#) Don T. and [Solberg](#) James J.
2. *Operations Research: An Introduction* H. A. Taha, 7th edition Prentice Hall of India Pt. Ltd. New Delhi.
3. *Introduction to Operations Research*, Hillier F. S. and Lieberman G. J, McGraw Hill International Edition.

MMT-E307: Topics in Graph Theory

Unit I

Planar graphs, Kuratowski's two graphs, Embedding on a sphere, Euler's formula, Kuratowski's theorem, Geometric dual of a planar graph, Independent sets, Coverings and Matchings, Factors, antifactory sets, f-factor theorem, Degree factors, (g, f) and $[a, b]$ -factors with examples. Basic equations, Matchings in bipartite graphs, Perfect matchings, Greedy and approximation algorithms,

Unit-II

Cut vertices and cut edges and their properties. Cut-sets and their properties, Vertex connectivity, Edge connectivity, Whitney's theorem, Manger's theorem (vertex and edge form), properties of a bond, Block graphs, Prove that every minimal cut is a bond and every Bond is a minimal cut.

Unit-III

Graph Matrices: Incidence and Adjacency matrix, Modified incidence matrix A_f , cycle matrix $B(G)$, Fundamental cycle matrix B_f , cut-set matrix $C(G)$, fundamental cut set matrix C_f , relation between A_f , B_f and C_f , path matrix, Matrix Tree Theorem, Eigen values of adjacency matrix, Energy of a graph.

Unit-IV

Digraphs, Types of digraphs, Types of connectedness, Euler Digraphs, Hamiltonian Digraphs, Arborescence, matrices in digraphs, Camions theorem, Tournaments, Score Structure in Digraphs, Characterization of score sequences, Landau's theorem, Oriented graphs and Avery's theorem,

Text Books:

1. Graph Theory, F. Harari, Addison-Wesley.
2. Graph Theory with Applications to Engineering and Computer Sciences, Narsingh Deo, Prentice Hall, India Ltd.

References:

1. Introduction to Graph Theory, D.B. West, PHI.
2. A First book at Graph Theory, J. Clark and D.A Holton, World Scientific.
3. Discrete Mathematics, J. Matousek and J. Nešetřil, Oxford University Press.

MMT-E308: Topics in Ring Theory

Unit-I

Definitions and simple properties, imbedding in a ring with unity, division rings and fields, Ideals and homomorphisms, prime ideals in commutative rings. Algebras, power series rings, Direct sums, sums and direct sums of ideals, ideal products and Nilpotent ideals, some conditions on ideals and ideals in complete matrix rings.

Unit-II

Subdirect sums of rings and sub directly irreducible rings. Boolean rings and their properties. Prime ideals and the prime radical, semiprime ideals. The prime radical of a ring, prime rings, the descending chain conditions and the prime radical.

Unit-III

Rings of Endomorphisms, the irreducible rings of endomorphisms, R-modules and rings of endomorphisms, irreducible rings and vector spaces. Dense rings of linear transformations, subspaces and descending chain conditions, some properties of the ring D_n , The Wedderburn-Artin Theorem.

Unit-IV

The Jacobson Radical: preliminary concepts, definition and simple examples, further properties of the Jacobson radical, the descending chain condition, idempotents and regular elements of dense rings, some further results on dense rings, another radical of a ring.

Text Book:

1. *The Theory of Rings*, Neal H. McCoy, Chelsea, publishing company.
2. *Rings and ideals*, Neal H. McCoy, The Carus Mathematical Monographs.

References:

1. *Noncommutative Rings*, I.N. Herstein The Carus Mathematical Monographs.
2. *Introduction to Noncommutative Algebra*, Matej Brešar, Springer.

MMT-E309: Calculus of Variation and Integral Equations

Unit 1

Introduction, problem of brachistochrone, problem of geodesics, isoperimetric problem, Variation and its properties, functions and functionals, Euler –Langrange equation, Necessary and Sufficient conditions for extrema. Comparison between the notion of extrema of a function and a functional.

Unit 2.

Variational problems with the fixed boundaries, Euler's equation, the fundamental lemma of the calculus of variations, examples, Functionals in the form of integrals, special cases containing only some of the variables, examples, Functionals involving more than one dependent variables and their first derivatives, the system of Euler's equations, Functionals depending on the higher derivatives of the dependent variables, Euler- Poisson equation, examples, Functionals containing several independent variables, Ostrogradsky equation, examples, Variational problems in parametric form, applications to differential equations, examples, Variational problems with moving boundaries, pencil of extremals, Transversality condition, examples.

Unit 3.

Introduction and basic examples, Classification, Conversion of Volterra Equation to ODE, Conversion of IVP and BVP to Integral Equation, Decomposition, direct computation, Successive approximation, Successive substitution methods for Fredholm Integral Equations, Volterra Integral Equation of first kind, Integral Equations with separable Kernel,

Unit 4.

Fredholm's first, second and third theorem, Integral Equations with symmetric kernel, Eigen function expansion, Hilbert-Schmidt theorem, Fredholm and Volterra Integro-Differential equation, Singular and nonlinear Integral Equation.

Text Books

1. *Calculus of variation*, M Gelfand and S V Fomin, Dover.
2. *Integral equations*, F G Tricomi, Dover.
3. Elsgolc, L.E. *Calculus of Variations*, Pergamon Press Ltd., 1962.
4. Corduneanu, C. *Integral Equations and Applications*, Cambridge University Press, 1991.

References:

1. *Calculus of variations with applications to physics*, R Weinstocks, Dover.
2. *Introduction to non linear differential equations and integral equations*, HL Davis, Dover.

MMT-E310: Fourier Analysis

Unit-1

• Fourier Series, Fourier Sine series and Fourier Cosine series, Smoothness, the Riemann-Lebesgue Lemma, the Dirichlet and the Fourier kernels, Area under Dirichlet kernel on $[0, \pi]$, the Riemann-Lebesgue property of the Dirichlet kernel, Continuous and Discrete Fourier kernel, point-wise Convergence of Fourier series, Criterion for point-wise convergence, Riemann's-Localization principle, Dini's test, Lipschitz's test.

Unit-2

• Pointwise convergence of Fourier Series, Selector property of $[\sin(n + \frac{1}{2} u/u)]$, Dirichlet point-wise convergence theorem, the Gregory series, Selector property of $(\sin w)/t$, point-wise convergence for boundary value, uniformly convergent trigonometric series and Fourier series, Convergent coefficients, Uniform convergence for piecewise smooth functions. The Gibb's phenomenon, the Gibb's phenomenon for a step function, Divergent Fourier series.

Unit-3

• Term-wise integration and term-wise differentiation, Trigonometric vs. Fourier series, Smoothness and speed of convergence, Dido's Lemma, other kinds of summability, Toeplitz summability, Abel summability, Fejer Kernel, properties of Fejer Kernels, Fejer's Theorem, A- summability of Fourier series, Hardy-Landau Theorem, the smoothing effect of $(C, 1)$ summation, Lebesgue point convergence Theorem.

Unit-4

• The finite Fourier transform, Convolution on the circle group T , the exponential form of Lebesgue theorem, the Fourier transform and residue, the Fourier map, Convolution on R , Inversion, Exponential form and Trigonometric form, $(C, 1)$ summability for integrals.

Textbooks:

1. *Fourier and Wavelet Analysis*, George Bachman, Lawrence Narici and Edward Beckenstein, Springer.

Reference books:

1. *Theory and applications of Fourier Analysis*, C S Rees, S M Shah, C V Stanojevic, Marcel Dekkar Inc., New York
2. *Fourier series*, Rajendra Bhatia, Hindustan Book Agency, Delhi.
3. *Walsh Functions and their applications*, K G Beauchhamp, Academic Press.
4. *The Fast Fourier Transform*, E O Brigham, Prentice Hall of India.

MMT-E402: Functional Analysis-II

Unit I

Weak and weak* topologies on a Banach space, Dual of a Banach space in its weak* topology, Goldstine's theorem, Banach-Alaoglu theorem and its simple consequences, Reflexivity of Banach spaces and weak compactness, Geometric and separation forms of Hahn-Banach Theorem, Applications of Hahn-Banach Theorem to: Banach limits, Markov-Kakutani theorem for a commuting family of maps, Dual of Subspace, Quotient space of a normed space.

Unit II

Complemented subspaces of Banach spaces, Dixmier's theorem on the complementability of the dual of a Banach space in its bidual, uncomplementability of C_0 in ℓ_∞ and its consequences, Banach's closed range theorem, characterizations of injective and surjective bounded linear mappings between Banach spaces.

Unit III

Completeness of L_p [a, b], Duals of ℓ_∞ and L_p spaces, Mazur-Ulam theorem on isometries between real normed spaces, applications of fundamental theorems of functional analysis to Radon-Nikodym Theorem, non-surjectivity of Laplace transform and Muntz theorem for $C[0,1]$.

Unit IV

ℓ_∞ and $C[0,1]$ as universal separable Banach spaces, l_1 as a quotient universal separable Banach space. Extreme points, Krein-Milman theorem and its simple consequences, Banach Stone Theorem on isometries between $C(X)$ and $C(Y)$.

Text Books:

1. *A Course in Functional Analysis*, J.B. Conway, Springer Verlag.
2. *Introduction to Banach Spaces and their geometry*, B. Beauzamy, North Holland.
3. *Functional Analysis*, W. Rudin, Tata McGraw Hill.
4. *An Introduction to Banach Space Theory*, R. E. Megginson, Springer Verlag.

References:

1. *Linear Analysis*, Ballobas. B, Camb. Univ. Press.
2. *Introduction to Banach Spaces and their geometry*, Beauzamy, B, North Holland.
3. *Infinite Dimensional Analysis*, C. D. Aliprantis and K C Border, Springer.
4. *A first course in functional Analysis*, C. Goffman and G. Pedrick, Prentice Hall.

MMT-E403: Commutative Algebra

Unit-I:

Recollection and Preliminaries of rings, ideals, ring homomorphisms, quotient rings, Nilpotent elements, units, prime and maximal ideals, Nilradical and Jacobson radical, local rings, operation on ideals and colons, extension and contraction of ideals, prime spectrum of a ring and Zariski topology. Direct sum and direct product of modules. Finitely generated and free modules, Nakayama's Lemma.

Unit-II

Exact, short exact and split exact sequences, Five Lemma, short Five Lemma and Snake lemma, categories and Functors, exact functors, the functor Hom, direct and Inverse Limits. Tensor product of modules, commutativity, Associativity and Exactness properties of tensor products, restriction and Extension of scalars.

Unit-III

Algebras, tensor product of algebras. Rings and modules of fractions, their local and exactness properties. Graded rings and modules. Homogeneous prime and maximal ideals, tensor algebras, symmetric algebras, exterior algebras. Anticommutative and Alternating algebras.

Unit-IV

Chain conditions, composition series for modules, Modules of finite length, Noetherian rings and modules Artinian rings and modules locally free modules, Primary decomposition, and related results. Support of a module, Dimension.

Text book:

1. *Introduction to Commutative Algebra*, M. F. Atiyah and I. G. Macdonald, Addison Wesley.

References:

1. *Basic Commutative Algebra*, Bulwant Singh, World Scientific.
2. *Commutative Algebra I, II, Zariski* and Sameul, Graduate texts in Mathematics Springer.
3. *Lectures on Modules and Rings*, T Y Lam, Graduate texts in Mathematics Springe

MMT-E404: Number Theory-II

Unit I

Power Residue: Quadratic residue and non-residues, Quadratic reciprocity law, The Legendre symbols, Definition and basic properties, Euler's Criterion, Gauss lemma. Quadratic reciprocity law, Characterization of primes for which 2, -2, 3, -3, 5, 6 and 10 are quadratic residues or non-residues.

Unit II

Jacobi symbol and its properties. The law of reciprocity for Jacobi symbols. Arithmetic functions, multiplicative functions, definition and basic examples, the function $[x]$ and its properties, the symbols γ , τ , σ , ϕ , the Mobius function, Mobius inversion formula.

Unit III

Simple continued fractions, Applications of the theory of infinite continued fractions to the approximations of irrationals by rationals, Hurwitz theorem and its sharpness. Irrationality of e and π , relation between Riemann zeta function and the set of primes, Characters, the orthogonality relation of characters, Dirichlet characters, the Dirichlet L-function and its properties, Dirichlet's theorem.

Unit IV

Polynomials, Division Algorithm for polynomials, irreducible polynomials, primitive polynomials, Gauss lemma, Schroeder-Einstein criteria for irreducibility, Algebraic Number and Algebraic integers, minimal polynomial of an algebraic numbers and Algebraic integers, minimal polynomial of an algebraic number Algebraic number field, Primes in Quadratic forms, Euclidean Quadratic forms, unique factorization, EQF has the UFP, solutions of $x^2 + y^2 = z^2$ and $x^3 + y^3 = z^3$ in rational integers.

Text Books:

1. *Topics in number theory*, W. J. Leveque, Vol. I, Addison Wesley publications, company, NIC.
2. *An introduction to the theory of numbers*, I. Niven & H.S Zuckerman.

References:

1. *Number theory*, Beovich and I. R. Sharfarich, Academic press.

MMT-E405: Theory of Semigroups

UNIT-I

Basic definitions and examples of semi-groups and sub semi groups, Null semigroup, Left/right zero semigroup, Nilpotent semigroup and nil semigroup. Ideal of a semigroup, Direct products, homomorphisms and Monogenic semigroups, Full transformation semigroups, Full partial transformation semigroup and Rectangular bands. Partial orders, semilattices and lattices, Congruences and quotients, Generating equivalences and congruences, Homomorphism theorems. Sub direct products.

Unit-II

Semigroup Actions, Free semi groups and Free Monoids, Universal property of Free semigroups, Semigroup presentations. Structure of semigroups: Green's relations, Simple and 0-simple semigroups, D-class structure, Inverses and D-classes.

Unit-III

Regular semigroups: Completely 0-simple semigroups, Ideal of completely 0-simple semigroups, Completely simple semigroups, Completely regular semigroups, Homomorphisms. Inverse semigroups: Equivalent characterizations, Vagner-Preston theorem, The natural partial order, Clifford semigroup, Free inverse semigroups.

UNIT-IV

Systems and bi-systems, Subsystems, Tensor product of systems, Existence and uniqueness of tensor product of bi-systems, Free products of semigroups, Uniqueness of free products in categorical sense, semigroup amalgams and amalgamated free products, Semigroup dominions and Isbell zigzag Theorem with examples and related results.

Text Book:

1. *Fundamentals of Semigroup Theory*, John M Howie, Clarendon press. Oxford.

References:

1. *The Algebraic Theory Semigroups, Vol. 1 and 2*, A H Clifford and G B Preston, Mathematical surveys of the AMS- 1961 and 1967.
2. *Techniques of Semigroup Theory*, P M Higgins, Oxford University Press.

MMT-E406: Probability and Measure

UNIT I

Probability measure, Construction of extension probability measure, $\pi - \lambda$ Theorem, Monotone class,

Probability space, Kolmogorov's probability model with examples, Random variables and random vectors, the change of variable formula, Moment generating functions and its properties, Marko's inequality, Cauchy's Shewartz inequality, Minkowski inequality, Stochastic process with examples and related results, Kolmogorov's consistency Theorem,

UNIT II

Product sigma algebra, Projection map and its properties, Cylinder set. Independent events and random variables, Boral Cantelli lemma. Tail sigma algebras Kolmogorov zero one law

UNIT III

Law of large numbers: Weak law of large numbers and proof of Bernstein Theorem using weak law of large numbers, Strong law of large numbers, Borel's strong law of large numbers, Glivenko Cantelli Theorem, Scheffe's Theorem, Etemadi's SLLN , Series of independent random variables, Levy's inequality, Kolmogorov's Three Series Theorem,

UNIT IV

Kolmogorov's and Marcinkiewz -Zygmunt SLLN's: Abel's summation formula, Marcinkiewz Zygmunt SLLNs for independent random variables that are not necessary identically distributed. Renewal theory: Renewal process, Stopping time, Wald's equation, Renewal Theorems and Renewal Equations.

Text Books:

1. *Probability and Measure*, Patrick, Billingsley, Wiley.
2. *Measure and Probability*, Athreya and Lahiri, CRC Press Inc.

References:

3. *Probability Theory and Examples*, Richard Durrett,
4. *A Course in Probability Theory*, Kai Lai Chung,
5. *An Introduction to Probability Theory and Applications Vol I, II*, W. Feller,
6. *Measure Theory and Probability Theory*, Athreya, Sunder, Springer.

MMT-E407: Wavelet Analysis

Unit-I

The Fejer Lebesgue inversion theorem, Convergence Assistance, Approximate identity, transforms of derivative and integrals, Fourier sine and cosine transforms Parseval's identities, the L_2 Theory, and the Plancheral Theorem.

Unit-II

Fourier transformations in $L^2(\mathbb{R})$: A Sampling Theorem, The Discrete and Fast Fourier Transforms.

Definitions, the DFT in Matrix form, Inversion Theorem for DFT, DFT Map, Fast Fourier Transform for $N = 2^k$, Buneman's Algorithm. FFT for $N = RC$, FFT Factor form.

Unit-III

Wavelets: Definition and examples, orthonormal basis from one function, multiresolution Analysis, Orthonormal Direct Sums, Mother Wavelets, From MRA to Mother wavelet.

Unit-IV

Construction of a scaling function with compact support, Shannon wavelets, Riesz bases and MRAs, Franklin wavelets, Frames, Weyl-Heisenberg Frames, Splines, The Continuous Wavelet Transforms.

Text books:

1. *Fourier and Wavelet Analysis*, G. Bachman, L. Narici and E. Beckenstein, Springer.

Reference books:

1. *An Introduction to Wavelet Analysis*, D. F. Walnut, Birkhauser.
2. *Wavelets: A Tutorial in Theory and Applications*, C. K. Chui, Academic Press, Boston, MA.
3. *A First Course on Wavelet Analysis*, E. Hernandez and G. Weiss, CRC Press.
4. *Applied Functional Analysis*, A. H. Siddiqui, Marcel-Dekker, New York.

MMT-E408: Topics in Analytic Theory of Polynomials

Unit I

Introduction, The fundamental theorem of algebra, (Revisited) Symmetric polynomials, The Continuity theorem, Orthogonal Polynomials, General Properties, The Classical Orthogonal Polynomials, Harmonic and Sub Harmonic functions, Tools from Matrix Analysis.

Unit II

Critical points in terms of zeros, Fundamental results and critical points, Convex Hulls and Gauss-Lucas theorem, some applications of Gauss Lucas theorem. Extensions of Gauss-Lucas theorem, Average distance from a line or a point Real polynomials and Jensen's theorem, Extensions of Jensen's theorem.

Unit III

Derivative estimates on the unit interval, Inequalities of S. Bernstein and A. Markov, Extensions of higher order derivatives, two other extensions, Dependence of the bounds on the zeros, some special classes, L_p analogous of Markov's inequality.

Unit IV

Coefficient Estimates, Polynomials on the unit circles. Coefficients of real trigonometric polynomials, Polynomials on the unit interval.

Text Books:

1. *Analytic theory of Polynomials*, Q.I. Rahman and G. Schmeisser.
2. *Geometry of polynomials*, Morris Marden.

References:

1. *Topics in polynomials: extremal properties, problems, inequalities, zeros*, G.V. Milovanovic, D.S. Mitrinovic and Th. M. Rassias.
2. *Problems and theorems in Analysis II*, G. Polya and G. Szego, Springer

MMT-E409: Banach Algebras

Unit-I

Banach Algebra: - Preliminaries on Banach Algebra's Invertible elements, the spectrum, spectral radius and the spectral radius formula, Gelfand- Mazur theorem, Gelfand mapping, maximal ideal space and its characterization, continuity of multiplicative functionals on a Banach algebra.

Unit-II

B^* Algebra and the Gelfand Naimark Theorem, Ideals in $C(X)$ and application to stone-Cech compactification and Banach stone theorem, structure of commutative C^* - Algebras.

Unit-III

Compact operators in Banach spaces, spectral theorem for compact Hermitian operators, spectral theorem for compact normal operators and its consequences.

UNIT-IV

Invariant subspace problem and its validity for compact Hermitian operators, Lomonosov's theorem on the existence of invariant subspaces for operators commuting with compact operators.

Text Books:

1. *A course in Functional Analysis*, J.B. Conway, (GTM 96, Springer Verlag).

References:

1. *Beginning Functional Analysis*, K. Saxe, Springer.
3. *Abstract Harmonic Analysis-I*, E. Hewitt & K.A Ross.

MMT-E410: Coding Theory

Unit-I

Block codes, codes and Hamming distance, linear codes, Generator matrix and systematic encoding, parity check matrix, hamming and simplex codes, dual codes. Decoding and error probability, equivalent codes,

Unit-II

Code constructions, product codes, concatenated codes. Bounds on codes: singleton bound, Griesmer bound, Hamming or sphere packing bound, and Plotkin bound Gilbert and Varshamov bounds, asymptotically good codes, weight enumerator and error probability.

Unit-III

Decoding complexity and erasures, information and covering set decoding. Cyclic codes, cyclic codes as ideals, generator polynomial, encoding cyclic codes, reversible codes, parity check polynomial. Polynomial codes.

Unit-IV

Structure of finite fields, cyclotomic polynomials, zeros of the generator polynomial. Bounds on the minimum distance: BCH bound, Hamming, simplex and Golay codes as cyclic codes. Locator polynomials and decoding cyclic codes: Mattson-Solomon polynomial, Newton identities, APGZ-algorithm, closed formulas.

Text Books:

1. *Error Correcting Codes*, Ruud Pellikan, Xin-Wen Wu, Stanislav Bulygin, and Relinde Jurrius, Cambridge University Press.
2. *Introduction to Coding Theory*, J. H. Van Lint, Springer-Verlag.
3. *Algebraic Coding Theory*, E. R. Berlekamp, Aegon Park Press, Laguna Hills CA.

References:

1. *Theory and practice of error control codes*, R.E. Blahut, Addison-Wesley.
2. *Algebraic codes for data transmission*, R.E. Blahut, Cambridge University Press.
3. *Handbook of coding theory*, V.S. Pless and W.C. Human, vol I and II, Elsevier Sc. Publ., Amsterdam.

MMT-E411: Complex Dynamics

Unit-I

Iteration of a Mobius transformation, attracting, repelling and indifferent fixed points. Iterations of $R(z) = z, z^2+c, z +$. The extended complex plane, chordal metric, spherical metric, rational maps, Lipschitz condition, conjugacy classes of rational maps.

Unit-II

Valency of a function, fixed points, Critical points, Riemann Hurwitz relation. Equicontinuous functions, normality sets, Fatou sets and Julia sets completely invariant sets, Normal families and equicontinuity.

Unit-III

Properties of Julia sets, exceptional points backward orbit, minimal property of Julia sets. Julia sets of commuting rational functions, structure of Fatou set, Topology of the Sphere, completely invariant components of the Fatou set.

Unit-IV

The Euler characteristic, Riemann Hurwitz formula for covering maps, maps between components of the Fatou sets, the number of components of Fatou sets, components of Julia sets.

Text Book:

1. *Iteration of rational functions*, A. F. Beardon, Springer.

Reference books:

1. *Complex dynamics*, L. Carleson and T. W. Gamelin, Springer.
2. *Holomorphic dynamics*, S. Morosawa, Y. Nishimura, M. Taniguchi, T. Ueda, Cambridge University Press.
3. *Dynamics of transcendental functions*, X. H. Hua, C. C. Yang, Gordon and Breach Science Pub.
4. *Dynamics in one complex Variable*, John Milnor, Annals of Mathematics Studies, Princeton University Press.

MMT-E412: Fluid Dynamics

Unit I

Physical Properties of fluids. Concept of fluids, Continuum Hypothesis, density, specific weight, specific volume, Kinematics of Fluids: Eulerian and Lagrangian methods of description of fluids, Equivalence of Eulerian and Lagrangian method, General motion of fluid element, integrability and compatibility conditions.

Unit-II

Strain rate tensor, stream line, path line, streak lines, stream function, vortex lines, circulation. Stresses in Fluids: Stress tensor, symmetry of stress tensor, transformation of stress components from one co-ordinate system to another, principle axes and principle values of stress tensor

Unit-III

Conservation Laws: Equation of conservation of mass, equation of conservation of momentum, Navier Stokes equation, equation of moments of momentum, Equation of energy, Basic equations in different co-ordinate systems, boundary conditions.

Unit IV

Irrotational and Rotational Flows: Bernoulli's equation, Bernoulli's equation for irrotational flows, Two dimensional irrotational incompressible flows, Blasius Theorem, Circle theorem, sources and sinks, sources sinks and doublets in two dimensional flows, methods of images.

Text Books:

1. *An introduction to fluid dynamics*, R.K. Rathy, Oxford and IBH Publishing co.

References:

1. *Theoretical Hydrodynamics*, L. N. Milne Thomson, Macmillan and Co. Ltd.
2. *Textbook of fluid dynamics*, F. Chorlton, CBS Publishers, Delhi.
3. *Fluid Mechanics*, L. D. Landau and E.N. Lipschitz, Pergamon Press, London

MMT-E413: Category Theory

Unit-I

Category; definition and examples, duality principle for categories, functors their types and properties, categories of categories, subcategories and their types, concrete categories and concrete functors, concrete subcategories and related results.

Unit-II

Transportability of concrete categories. Natural transformations, functor categories, Galois correspondence, initial and terminal objects, sections and retractions, monomorphisms and epimorphisms, equalizers and co-equalizers.

Unit-III

Sub objects and quotient objects, pointed categories, discrete and indiscrete objects, embeddings and quotient morphisms, universal arrows and free objects. Sources and sinks, products, categories with products, sources in concrete categories, concrete products, co-products, examples of co-products.

Unit-IV

Pullbacks and pushouts, properties of pullbacks and pushouts, limits, preservation of limits, co-limits, completeness and co-completeness, functors and limits.

Text Book:

1. *Abstract and Concrete Categories the Joy of Cats*, Jiri Adamek, Horst Herrlich George E. Streckers. Dover Books on Mathematics

References:

1. *Category Theory*, Steve Awody Oxford Logic Guides.
2. *Categories for working mathematicians*, Sundar Maclane Graduate texts in Mathematics.
3. *An Introduction to the language of Category Theory*, Steven Roman, Birkhauser.

