Course Structure and Syllabus for M.A/M.Sc Mathematics

YEAR AND BATCH: 2020 ONWARDS

Department of Mathematics
Central University of Kashmir
Ganderbal

Contents

M.A./M.Sc. Mathematics
Revised Syllabus

Semester-I

Course Structure				Marks			
S.No.	**Course Code**	**Course Title**	**Credits**	**CIA**	**MT**	**ET**	**TOTAL**
1	MMT-C101	Abstract Algebra-I	4	20	30	50	100
2	MMT-C102	Real Analysis-I	4	20	30	50	100
3	MMT-C103	Complex Analysis-I	4	20	30	50	100
4	MMT-C104	Topology	4	20	30	50	100
5	MMT-C105	Number Theory	4	20	30	50	100
6	SS	Soft Skills(open elective)	4	20	30	50	100
Total			24	120	180	300	600

Semester I

MMT-C101 – Abstract Algebra-I

Unit-I

Group actions, stabilizers and orbits, Cayley's theorem, class equation and its applications, Conjugacy in S_n and cycle decomposition. Simple groups, simplicity of alternating groups. Automorphisms: examples and related results, inner automorphism and related results.

Unit-II

Finite abelian groups: Direct sums and direct products, free abelain groups and their freeness property, p-primary groups, primary decomposition, pure subgroups, Basis Theorem, Fundamental theorem of finite abelian groups and its applications.

Unit-III

Cauchy's theorem of finite groups, Sylow theorems and their applications: groups of order pq, p and q are primes and $p < q$, groups of order $30, 12, 60$ and groups of order p^2q where p and q are distinct primes. Projective unimodular groups, free groups and presentations.

Unit-IV

Maximal subgroups and p-groups, nilpotent groups exampls and related results, commutators and lower central series, solvable groups and derived series, composition series for groups, Zessenhaus's Lemma, Schreier Refinement Theorem and Jordan Holder Theorem.

Recommended books:

1. Advanced Modern Algebra, Joseph J. Rotman, Graduate studies in Mathematics, AMS.
2. Abstract Algebra, D. S. Dumit and R. M. Foote, Wiley.
3. Topics in Algebra, I. N. Herstein, Wiley.
4. Algebra, M. Artin, Pearson.

MMT-C102 – Real Analysis-I

Unit-I

Review of functions continuous on intervals: boundedness theorem, maximum-minimum theorem, intermediate value theorem; (statements only). Sequential definitions of continuity and uniform continuity. The Heine - Cantor theorem and the continuous extension theorem. Inequalities: Arithmetic Mean - Geometric mean inequality, Cauchy-Schwarz inequality, Chebyshev's inequality, Holder's and Minkowski's inequalities, Convex and concave functions, Jensen's inequality, Bernoulli's inequality, some applications involving inequalities.

Unit-II

Riemann Integration: Definition of the Riemann integral. Necessary and sufficient condition for existence of Riemann integral. The Darboux definition of an integral: upper and lower integrals. The Darboux integral and the integrability criterion. Equivalence of Darboux and Riemann integrability. Algebra of integrable functions. Riemann integrability of $|f|$ whenever f is Riemann integrable. Fundamental theorems and the mean value theorem of integral calculus.

Unit-III

Improper Integrals: Integration of unbounded functions with finite limit of integration, Comparison of tests for convergence of improper integrals, Cauchy's test for convergence. Absolute convergence, Infinite range of integration of bounded functions, convergence of integrals of unbounded functions with infinite limits of integration, integrated as a product of functions, Abel's and Dirichlet's tests of convergence.

Unit-IV

Review of conditional and absolute convergence of series. Dirichlet's and Reimann's Rearrangement theorems for series. Function spaces: Sequences and series of functions. The point-wise and the uniform convergence on an interval. Cauchy's criterion for uniform convergence. The M_n-test for uniform convergence of sequences. Weierstrass's M-Test, Abel's and Dirichlet's tests for uniform convergence of series. Uniform convergence and continuity. Uniform convergence and integrability. Uniform convergence and differentiability. Weierstrass's approximation theorem.

Recommended books:

1. Robert G. Bartle and Donald R. Sherbert, Introduction to Real Analysis, 4th Edition, Wiley India Pvt. Ltd.

2. Inequalities: An Approach Through Problems, S. Venketachela, (TRIM) Hindustan Book Agency.

3. R. Goldberg, Methods of Real Analysis, John Wiley and Sons, 1976.

4. T.M. Apostol, Mathematical Analysis, Narosa, 2004.

MMT-C103 – Complex Analysis-I

Unit-I

Functions of a complex variable, Limits, Continuity, Differentiability, Cauchy-Riemann Equations and their applications, Analytic function, Harmonic function, The functions like e^z, $\sin z$, $\cos z$ and the complex logarithm. Contour integral, Cauchy's theorem, Cauchy- Goursat's theorem, Cauchy's integral formula, Higher order derivatives, Morera's theorem, Cauchy's inequality, Liouville's theorem and its applications, Winding numbers-index of a point with respect to a closed curve.

Unit-II

Bilinear transformations- Properties and Classification, Fixed Points, Cross Ratios, Inverse Points and Critical Points. Conformal mapping, Mappings of: Upper half plane on to unit disc, Unit disc onto unit disc, left half plan on to unit disc, Circle onto circle. The transformations: $w = \sqrt{z},\ z^2,\ \frac{1}{2}(z + \frac{1}{z})$.

Unit-III

Power Series, Radius of convergence of a power series, Cauchy's-Hadamard formula for finding radius of convergence, Taylor theorem, Taylor's series, Expansion of analytic functions in a power series, Laurent's series, isolated and essential singularities. Behavior of functions at infinity, Casorati-Weirestrass's Theorem.

Unit-IV

Residues: Cauchy's residue theorem and its applications, Calculation of residues, Evaluation of definite integrals by the method of residues, Perseval's identity. Infinite products, Convergence and divergence of infinite products.

Recommended books:

1. L. Ahlfors, Complex Analysis, Tata McGraw Hill.
2. Richard Silverman, Introductory Complex Analysis, Dover Publications.
3. S. Ponnusamy, Foundations of Complex Analysis, Narosa.
4. Z. Nihari, Conformal Mappings, Dover Publications.
5. E.C. Titchmarsh, Theory of Functions, Oxford Science Publications.

MMT-C104 – Topology

Unit-I

Review of countable sets, uncountable sets and Schroeder-Bernstein theorem. Metric spaces: definition and examples, completeness in metric spaces, Baire's category theorem, completion of a metric space, Cantor's intersection theorem, uniformly continuous mappings with examples, Banach's contraction principle with applications to the inverse function Theorem in $\mathbb{R}$.

Unit-II

Topological spaces: open sets, discrete and indiscrete topologies, finite and cofinite topologies, coarser and finer topologies, basis for a topology, lower limit topology, the order topology, the product and the subspace topology, closed sets, closure and interior of a set, Hausdroff spaces: closed sets and convergence in Hausdroff spaces, metric topology.

Unit-III

Continuous functions, homeomorphisms, the pasting lemma. Connected spaces: separation, connected subspaces of the real line, intermediate value theorem, path connectedness, components and local connectedness. Compact spaces: finite intersection property, Extreme value theorem, Lebesgue covering lemma, Heine-Borel Theorem.

Unit-IV

Limit point compactness and local compactness, one point compactificationg. The countability axioms, Lindelöf's Theorem. Separation Axioms: T_1 spaces and Hausdroff spaces, regular and normal spaces, Urysohn Lemma, Urysohn Metrization Theorem, Tietze Extension Theorem, Tychnoff Theorem.

Recommended books:

1. Introduction to Topology and Modern Analysis, G. F. Simmons, Tata McGraw Hill Education.
2. Topology, J. R. Mukres, Pearson.
3. General Topology, J. L. Kelley, Springer.

MMT-C105 – Number Theory

Unit-I

Principle of mathematical inductions, Well ordering principle, Division algorithm, Divisibility in integers, GCD, its properties, GCD is the linear combination of two integers, relatively prime integers, Euclidean algorithm. Sequence of primes, Fundamental theorem of Arithmetic, LCM, EuclidÂ's lemma, infinitude of primes, no polynomial $f(x)$ with integers can be a prime for all x, arbitrary large gaps in the sequence of primes, DirichletÂ's theorem, Goldbach conjecture, Radix representation. Fermat numbers, Mersenne numbers.

Unit-II

Linear Diophantine equations, positive solutions. Congruences and properties, Arithmetic functions and multiplicative functions, basic examples. Euler's ϕ function and its properties. Complete Residue System, Reduced Residue System and related results. FermatÂ's little theorem, EulerÂ's theorem, WilsonÂ's theorem and their applications. Greatest integer function, properties and related results.

Unit-III

System of linear Congruences, Chinese Remainder theorem, solution of polynomial Congruences, a necessary and sufficient condition that the congruence $ax \equiv b(mod\ m)$ is solvable and related results, Congruences of higher order, Congruences with prime power, prime moduli and related results, LagrangeÂ's theorem.

Unit-IV

Division Algorithm for polynomials, Factor theorem and its generalization, equivalence of polynomials, form of degree n, Chevalley's theorem, Warning's theorem, Quadratic forms over a field of characteristics $\neq 2$, equivalent quadratic forms, Direct sum of quadratic forms, Witt's theorem, Representation of field elements, Hermite's theorem, an integer is the sum of two, three and four squares.

Recommended books:

1. Elementary number Theory, David M. Burton, McGraw Hill Education, 7th Edition.
2. An introduction to the Theory of Numbers, I. Niven, H.S. Zuckerman and H. L. Montgomery, Wiley; fifth Edition.
3. Topics in Number Theory, W. J. Leveque, Dover Publications; vols 1 and 2 edition.
4. An Introduction to the Theory of Numbers, G.H. Hardy and E. M. Wright, Oxford University Press; 6th Edition.
5. Elementary Number Theory with Applications, Thomas Koshy, Academic Press; 2nd Edition.

M.A./M.Sc. Mathematics
Revised Syllabus

Semester-II

Course Structure				Marks			
S.No.	**Course Code**	**Course Title**	**Credits**	**CIA**	**MT**	**ET**	**TOTAL**
1	MMT-C201	Abstract Algebra-II	4	20	30	50	100
2	MMT-C202	Real Analysis-II	4	20	30	50	100
3	MMT-C203	Complex Analysis-II	4	20	30	50	100
4	MMT-C204	Linear Algebra	4	20	30	50	100
5	MMT-C205	Graph Theory	4	20	30	50	100
6	SO	Social Orientation (open elective)	4	20	30	50	100
		Total	24	120	180	300	600

Semester II

MMT-C201 – Abstract Algebra-II

Unit-I

Review of rings their basic properties and examples: quadratic integer rings and their properties, matrix rings, polynomial rings and related results, power series rings. Integral domains fields and related results, algebra of ideals, prime ideals, maximal ideals and related results, quotient rings and isomorphism theorems on rings, Chinese remainder theorem.

Unit-II

Field of fractions and embedding theorem. Euclidean domains; examples and related results, existence of GCD and related results in Euclidean domains, universal side divisors and their properties. Principal ideal domains (PIDs) examples and related results, Dedekind-Hasse norm, PIDs which are not Euclidean domains.

Unit-III

Irreducible elements and primes with examples and related results, Unique Factorization Domains (UFDs) examples and related results; PID implies UFD but not conversely. Factorization in Gaussian integers: Fermat's theorem on sum of squares and its applications .

Unit-IV

Polynomial rings in severable variables, polynomial rings over fields. Unique factorization in polynomial rings: Gauss's Lemma and Gauss's Theorem. Irreducibility criteria for polynomials, Eisenstein's Criterion. Hilbert's Basis Theorem.

Recommended books:

1. Advanced Modern Algebra, Joseph J. Rotman, Graduate studies in Mathematics, AMS.
2. Abstract Algebra, D. S. Dumit and R. M. Foote, Wiley.
3. Topics in Algebra, I. N. Herstein, Wiley.
4. Algebra, M. Artin, Pearson.

MMT-C202 – Real Analysis-II

Unit-I

Partial derivatives, directional derivatives, total derivative continuity and their relationships for functions from : R^n to R. Matrix of a linear function and Jacobian of a differentiable function at a point, chain rule, mean value theorem for differentiable functions. Taylor's theorem for functions from R^n to R. Implicit and Inverse function theorems in R^n. Extremum problems for functions on R^n. Lagrange's multipliers, Multiple Riemann Integral and change of variables formula for multiple Riemann integrals.

Unit-II

Semirings, algebras and σ-algebras. Measure on a semiring, outer measure and measurable sets, Caratheodary extension of a measure, outer measure of an interval as its length. Lebesgue measurable and non-measurable sets, measurable functions and their characterizations, algebra of measurable functions, almost everywhere property.

Unit-III

Simple functions, step functions and integral of a step function. monotone and order countinuity of the integral of step functions. Lebesgue measure on R^n. Steinhauss theorem, Ostroviski's theorem on measurable solution of $f(x+y) = f(x)+f(y)$. Convergence a.e., convergence in measure, almost uniform convergence, their relationship on sets of finite measure, Egroff's theorem.

Unit-IV

Upper functions, integral of upper functions, lebesgue integrable functions, Levi's theorem, Fatous lemma, Monotone convergence theorem, Lebesgue dominated convergence theorem, the riemann integral as a lebesgue integral, Lebesgue-Vitali theorem.

Recommended books:

1. H.L. Royden, Real Analysis, Pearson.
2. Walter Rudin, Principles of Mathematical Analysis, 3rd Edition. McGraw Hill.
3. T. M. Apostol, "Mathematical Analysis-Second Edition, Narosa Publishing House.
4. D.V. Widder, Advanced Calculus. 2/e, Prentice Hall of India, New Delhi.

MMT-C203 – Complex Analysis-II

Unit-I

Maximum Modulus Principle, Schwarz Lemma and its generalization, Meromorphic function, Argument Principle, Rouche's theorem with application, Inverse function Theorem, Poisson integral formula for a circle and half plane, Poisson Jenson formula, Carleman's theorem, Hadamard three-circle theorem and the theorem of Borel and Carathéodory.

Unit-II

Principle of analytic continuation, uniqueness of direct analytical continuations and uniqueness of analytic continuation along a curve. Power series method of analytic continuation, Functions with natural boundaries and related examples. Schwartz reflection principle, functions with positive real part.

Unit-III

Space of analytic functions, Hurwitz's theorem, Montel's theorem, Riemann Mapping theorem, Weierstrass factorization theorem, Gamma function and its properties. Riemann Zeta function, Riemann's functional equation. Harmonic functions on a disc, Harnack's inequality and theorem, Dirichlet's problem.

Unit-IV

Canonical products, order of an entire functions, Exponential convergence, Borel theorem, Hadamard's factorization theorem, The Range of analytic functions, Bloch's Theorem, Schottky's Theorem, The Little Picard's Theorem, Landau's Theorem, Great Picard Theorem (statement and applications only), Univalent function. Bieberbach's conjecture (statement only) and the 1/4-theorem.

Recommended books:

1. Complex Analysis, L. Ahlfors, McGraw Hill Education.
2. Theory of Functions, E.C. Titchmarsh Oxford University Press.
3. Functions of one complex variable, J.B. Conway, Narosa.
4. Complex Analysis, Richard Silverman, Dover publications.
5. Theory of Functions of a Complex variable, A. I. Markushevish, AMS/Chelsea Publication.

MMT-C204 – Linear Algebra

Unit-I

Review of linear transformations, algebra of linear transformations, singular and non-singular transformations. Linear functionals and dual spaces, characteristic values, characteristic polynomials, minimal polynomials. Diagonalizability of linear operators, invariant sub spaces, simultaneous diagonalization, transpose of linear operator.

Unit-II

Inner product spaces (IPS): examples and basic properties. Gram-Schmidt orthogonalization, orthogonal complements and orthogonal projections. Adjoint operators, unitary, normal and Hermitian operators on IPS with examples and related results. Diagonalizability of linear operators on finite dimensional IPS. Bilinear forms.

Unit-III

Modules: basic definitions and examples, submodules, quotient modules and module homomorphisms, isomorphism theorems on modules, generation of modules and direct sums, free modules, torsion, torsion free modules annihilators, modules over PID's, rank of a module and related results.

Unit-IV

Fundamental theorem for finitely generated modules (existence and uniqueness), primary decomposition theorem, elementary divisors and invariant factors, rational canonical form,converting $n \times n$ matrix into rational canonical form, Jordan canonical form, converting $n \times n$ matrix into Jordan canonical form, changing one canonical form into another.

Recommended books:

1. Linear Algebra, Hoffman and Kunz, Pearson.
2. Abstract Algebra, D. S. Dumit and R. M. Foote, Wiley Sons.
3. University Algebra, N. S. Gopalakrishnan, New Age International Publishers.
4. Advanced Modern Algebra, Joseph J. Rotman, Graduate Studies in Mathematics.

MMT-C205 – Graph Theory

Unit-I: Introduction to Graphs
Types of graphs, graph isomorphism, walks, paths and cycles, Konig's theorem on bipartite graphs, graph operations, Konigsberg bridge problem, Euler graphs and Euler's theorem, Hamiltonian graphs, Dirac's theorem, Ore's theorem, Hamiltonian cycles in K_n (n odd), connectedness, signed graphs, balanced signed graphs.

Unit-II: Degree sequences and Trees
Degree sequences, Wang and Klietman theorem, Havil-Hakimi theorem, Hakimi's theorem, Erdos-Gallai theorem, Degree sets, trees and their properties, binary and spanning trees, complexity of a graph, Labelled trees, Cayley's Theorem, fundamental cycles.

Unit-III: Connectivity and Planarity
Cut sets, vertex and edge connectivity, Whitney's theorem, properties of a bond, block graphs, planar graphs, Euler's formulla, Kuratowski's theorem, geometric dual, Whitney's theorem, regular polyhedra.

Unit-IV: Graph Matrices
Incidence matrix $A(G)$, modified incidence matrix A_f, cycle matrix $B(G)$, fundamental cycle matrix B_f, cut-set matrix $C(G)$, fundamental cut set matrix C_f, relation between A_f, B_f and C_f , path matrix, adjacency matrix, matrix tree theorem, Laplacian Matrix.

Recommended books:

1. A Text Book of Graph Theory, R. Balakrishnan, Ranganathan, Springer- Verlag.
2. Extremal Graph Theory, B. Bollobas, Academic Press.
3. An Introduction to Graph Theory, S. Pirzada, Universities Press, Orient Blackswan, Hyderabad, India, (2012).
4. Introduction to Graph Theory, Gary Chartrand and Ping Zhang, Tata McGraw Hill.
5. Graph Theory with Applications to Engineering and Computer Science, Narsingh Deo, Prentice Hall.

M.A/M.Sc. Mathematics
Revised Syllabus

Semester-III

Course Structure				Marks			
S.No.	**Course Code**	**Course Title**	**Credits**	**CIA**	**MT**	**ET**	**TOTAL**
1	MMT-C301	Functional Analysis-I	4	20	30	50	100
2	MMT-C302	Differential Geometry	4	20	30	50	100
3	MMT-C303	Ordinary and Partial Differential equations	4	20	30	50	100
4	OGE	Open Generic Elective	4	20	30	50	100
Any Two of the Following							
5	MMT-E304	Commutative Algebra	4	20	30	50	100
6	MMT-E305	Theory of Semigroups-I	4	20	30	50	100
7	MMT-E306	Advanced Number Theory	4	20	30	50	100
8	MMT-E307	Complex Dynamics	4	20	30	50	100
9	MMT-E308	Lattices and Ordered Structures	4	20	30	50	100
10	MMT-E309	Topics in Ring Theory	4	20	30	50	100
11	MMT-E310	Algebraic Topology	4	20	30	50	100
12	MMT-E311	Topics in Applied Mathematics	4	20	30	50	100
Total			24	120	180	300	600

Semester III

MMT-C301 – Functional Analysis-I

Unit-I

Banach Spaces: Definition and examples, subspaces, quotient spaces, continuous linear operators and their characterization, completeness of the space $L(X, Y)$ of bounded linear operators (and its converse), incompleteness of $C[a, b]$, under the integral norm, finite dimensional Banach spaces, Equivalence of norms on finite dimensional space and its consequences, dual of a normed linear space, Hahn Banach theorem (extension form) and its applications, complemented subspaces, duals of $C_0, l_p\ (p \geq 1), C[a, b]$.

Unit-II

Uniform boundedness principle and weak boundedness, dimension of an n-dimensional Banach space, conjugate of a continuous linear operator and its properties, Banach-Steinhauss Theorem, Open Mapping and Closed Graph Theorems, counterexamples to Banach-teinhauss, Open Mapping Theorem and Closed Graph Theorems for incomplete domain and range spaces, separable Banach spaces and the separability of some concrete Banach spaces $(C_0, C[0, 1], l_p, p \geq 1)$, reflexive Banach Spaces, closed subspace and the dual of a reflexive Banach space, examples of reflexive and non-reflexive Banach spaces.

Unit-III

Hilbert spaces: Definition and examples, Cauchy's Schwartz inequality, Parallelogram law, orthonormal systems, Bessel's inequality and Parseval's Identity for complete orthonormal systems, Riesz-Fischer Theorem, Gram Schmidt process, orthonormal basis in separable Hilbert spaces.Fourier Series with respect to an orthonormal basis.

Unit-IV

Projection Theorem, Riesz Representation Theorem. counterexample to the Projection theorem and Riesz Representation Theorem for incomplete spaces, Hilbert property of the dual of a Hilbert space and counterexamples for incomplete inner product spaces, reflexivity of Hilbert space, adjoint of a Hilbert space operator, weak convergence and Bolzano-Weirstrass property in Hilbert spaces, normal and unitary operators, finite dimensional spectral theorem for normal operators.

Recommended books:

1. Introduction to Topology and Modern Analysis, G. F. Simons, Tata McGraw Hill Education.
2. Functional Analysis, B.V.Limaya, New age International Publishers.
3. A First Course in Functional Analysis, C.Goffman G. Pedrick, PHI.
4. Elements of Functional Analysis, L.A. Lusternick & V.J. Sobolov, Hindustan Publishing Corporation.
5. A Course in Functional Analysis, J.B. Conway Springer.

MMT-C302 – Differential Geometry

Unit-I

Curves, Planar curves, Regular curves, Parameterization of curves, Arc-length, and arc-length is independent of parameterization, unit speed curves. Plane curves: Curvature of plane curves, osculating circle, centre of curvature. Computation of curvature of plane curves. Directed curvature, Fundamental theorem for plane curves.Examples: Straight line, circle, ellipse, tractrix, evolutes and involutes. Space curves: tangent vector, unit normalvector and unit binormal vector to a space curve. Curvature and torsion of a space curve. The Frenet–Serret Theorem. First Fundamental theorem of space curves. Intrinsic equation of a curve. Computation of curvature andtorsion.

Unit-II

A brief review of continuity and differentiability and revisiting Inverse function theorem (statement only), Surfaces, Regular surfaces, coordinate charts. Change of parameters; Differentiable functions on surfaces, The tangent plane, Differential of a map, First Fundamental Form, Orientation of surfaces, line element and its invariance under change of coordinates, angle between two curves. Geometrical definition of area.

Unit-III

Curvature of a Surface: Normal curvature, Euler's work on principal curvature, Qualitative behaviour of a surface near a point with prescribed principal curvatures. The Gauss map and its differential, together with its properties. Second fundamental form. Gaussian curvature, Mean curvature Weingarten equation. Ruled surfaces and minimal surfaces, Isothermal coordinates. Surfaces with constant positive or negative Gaussian curvature. Gaussian curvature in terms of area. Line of curvature, Rodrigues's formula for line of curvature, Equivalence of Surfaces: Isometry between surfaces, local isometry, and characterization of local isometry.

Unit-IV

Christoffel symbols. Expressing Christoffel symbols in terms of metric coefficients and their derivative. Theorema Egregium (Gaussian curvature is intrinsic). Isometric surfaces have same Gaussian curvatures at corre- sponding points. Gauss equations and Manardi Codazzi equations for surfaces. Fundamental Theorem for regular surface. (Statement only). Geodesics: Geodesic curvature, Geodesic curvature is intrinsic, Equations of Geodesic, Geodesic on surfaces of revolution and sphere. Geodesic as distance minimizing curves. Gauss-Bonnet theorem (statement) and its implication for geodesic triangle on sphere.

Recommended books:

1. Elementary Differential Geometry, Andrew Pressly, Springer.
2. Elementary Differential Geometry, Barret O'Neil, Academic Press.
3. Geometry from a Differentiable Viewpoint, John Mc Cleary, Cambridge Univ. Press.

MMT-C303 – Ordinary and Partial Differential Equations

Unit-I

Initial value problems (IVP) of first order ODE, Ascoli-lemma, Fundamental theorem for the existence and uniqueness of solution of an IVP, Picard's theorem on the existence and uniqueness of solutions to an IVP. Method of successive approximations. Strum-Lioville Problem (SLP) for second order differential equation, properties of eigenvalues and eigenfunctions of SLP. Series solution for ordinary differential equations of second order. Method of variation of parameters.

Unit-II

Linear dependence and independence of solutions of an ODE, properties of Wronskian of the solutions of an ODE and Abel's formula. Method of construction of Green's function of one dimension. Simultaneous equation $\frac{dx}{P} = \frac{dx}{Q} = \frac{dz}{R}$ and its solution by use of multipliers and a second integral found by the help of first. Total differential equations $Pdx + Qdy + Rdz = 0$. Necessary and sufficient condition for integrability of a total differential equation. Geometric interpretation of the $Pdx + Qdy + Rdz = 0$. Simultaneous system of ODE in matrix form.

Unit-III

Partial Differential Equations of first order, origins of first order PDEs, Cauchy Problem for first order equations, Linear equations of the first order, Nonlinear PDEs of the first order, Lagrange and Charpit's methods for solving first order PDEs, Classification of 2nd order PDE, General solution of higher order PDEs with constant coefficients, Method of separation of variables for three basic equations: Laplace, Heat and Wave equations.

Unit-IV

D-Alembert's solution of wave equation, the initial value problem in three space, Poisson's method of spherical averages, Hadamard's method of descent, Duhamel's Principle, the inhomogeneous wave equation. Mean value property and maximum principles for elliptic problems and maximum principle for heat equation. Green's function for half space and disc for the Laplace operator.

Recommended books:

1. G. F. Simmons: Differential Equations, Tata McGraw Hill.
2. E. A. Coddington and N. Levinson: Theory of Ordinary Differential Equations, Dover.
3. D. Somasundaram, Ordinary Differential Equations, Narosa Publishers.
4. Partial Differential Equations by T. Amarnath, Alpha Science.

MMT-E304 – Commutative Algebra

Unit-I

Rings, ring homomorphism, ideals, quotients, zero divisors, nilpotents and units. Prime and maximal and comaximal ideals, nilradical and Jacobsons radical. Operations on ideals, extension and contraction. Chinese remainder theorem and prime avoidance lemma.

Unit-II

Finitely generated modules, Nakayama lemma, exact sequences, free and projective modules, Tensor product of modules, flat modules. Restriction and extension of scalars. Rings and modules of fractions and localization.

Unit-III

Chain conditions, Noetherian rings. Hilbert basis theorem, primary decomposition in Notherian rings. Artinian rings, modules over PID. Modules of finite length and primary Decomposition.

Unit-IV

Integral dependence, Going-up and Going-down theorems, valuation rings, integral extensions, Hilbert's Nullstellansatz, Noether normalization theorem.

Recommended books:

1. An Introduction to Commutative Algebra, M. F. Atiyah and I. G. Macdonald, Addison-Wesley.
2. Introduction To Commutative Algebra And Algebraic Geometry, E. Kunz, Birkhäuser.
3. Cohen-Macaulay Rings, W. Bruns and J. Herzog, Cambridge University Press.
4. Commutative ring theory, H. Matsumura, Cambridge University Press.

MMT-E305 – Theory of Semigroups-I

Unit-I

Basic definitions and examples of semigroups. Direct product of semigroups, rectangular bands. Monogenic semigroups, ordered sets, semilattices and lattices. Binary relations and partial mappings, equivalences and congruences. Homomorphism Theorems. Generating equivalences and congruences.

Unit-II

Free semigroups and free monoids, semigroup presentations, ideals and Rees congruences, lattices of equivalences and congruences. Green's equivalences: Definitions, examples and related results. Green's lemma and related results.

Unit-III

Regular D-classes and related results, regular semigroups and related results, Lallement's Lemma. Simple and 0-simple semigroups, minimal and 0-minimal ideals. Inverse semigroups, symmetric inverse semigroups, Vagner-Preston Theorem..

Unit-IV

Systems, bi-systems and subsystems, tensor product of systems; its existence and uniqueness, some properties of tensor product. Free product of semigroups, free product as a coproduct. Semigroup amalgams and its embaddability, amalgamated free product of semigroups. Dominions and zigzags and their relation with semigroup amalgams, Isbell's Zigzag Theorem on dominions and its applications.

Recommended books:

1. Fudamentals of Semigroup Theory, J. M. Howie, Oxford Science Publications.
2. The Algebraic Theory of Semigroups, A. H. Clifford and G. B. Preston, American Mathematical Scociety.
3. Nine Chapters on the Semigroup Art, Alan J. Cain, Porto & Lisbon.

MMT-E306 – Advanced Number Theory

Unit-I

Number-theoretic functions, multiplicative functions, totally multiplicative functions, basic examples. Functions: $\tau(n)$, $\sigma(n)$ and their properties. Perfect numbers, necessary and sufficient condition for an even number to be perfect. Moebius function $\mu(n)$, Moebius inversion formula, sum of odd divisors of n, $\sum_{d|n} \mu(d)\tau(d)$, $\sum_{d|n} \mu(d)\sigma(d)$ and related results. Euler's constant γ, the symbols o, O and $\sim$.

Unit-II

Order of an element modulo(m), related results, if $d|p-1$, the congruence $x^d \equiv (mod\ p)$ has exactly d solutions. Primitive roots, primitive roots of an odd prime, primitive root of an odd prime with any power. Universal exponent $\lambda(m)$ of m, refinement of Euler's theorem, the numbers having primitive roots are $1, 2, 4, p^\alpha$ and $2p^\alpha$, where p is any odd prime. Quadratic residues and quadratic non-residues, Euler's criterion and related results.

Unit-III

Legendre symbol and its properties, lemma of Gauss, quadratic reciprocity law for Legendre symbol, characterization of primes for which $2, -2, 3, -3, 5$ are quadratic residue or quadratic non-residue. Jacobi symbol and its properties, quadratic reciprocity law for Jacobi symbol and other related results. Farey fractions, Farey sequences, rational approximations, their related results.

Unit-IV

Simple finite continued fractions, infinite continued fractions, irrational numbers, infinite continued fraction is irrational and related results. Irrationality of e and π, Hurwitz theorem and its sharpness. Character, orthogonality relation for characters, Riemann zeta function $\zeta(s) = \prod_p \left(1 - \frac{1}{p^s}\right)^{-1}$ for $s > 1$, $\frac{1}{\zeta(s)} = \sum_{m=1}^{\infty} \frac{\mu(m)}{m^s}$. Algebraic numbers and Algebraic integers, Algebraic number field, Primes in Quadratic forms, Euclidean Quadratic forms, unique factorization, the diophantine equations $x^2 + y^2 = z^2$ and $x^3 + y^3 = z^3$.

Recommended books:

1. Elementary number Theory, David M. Burton, McGraw Hill Education.
2. An introduction to the Theory of Numbers, I. Niven and H.S. Zuckermann, Wiley.
3. Topics in Number Theory, W. J. Leveque, Dover Publications.

MMT-E307 – Complex Dynamics

Unit-I

Iteration of a Mobius transformation, attracting, repelling and indifferent fixed points. Iterations of $R(z) = z, z^2 + c$. The extended complex plane, chordal metric, spherical metric, rational maps, Lipschitz condition, conjugacy classes of rational maps.

Unit-II

Valency of a function, fixed points, Critical points, Riemann Hurwitz relation. Equicontinuous functions, normality sets, Fatou sets and Julia sets completely invariant sets, Normal families and equicontinuity.

Unit-III

Properties of Julia sets, exceptional points backward orbit, minimal property of Julia sets. Julia sets of commuting rational functions, structure of Fatou set, Topology of the Sphere, Completely invariant components of the Fatou set.

Unit-IV

The Euler characteristic, Riemann Hurwitz formula for covering maps, maps between components of the Fatou sets, the number of components of Fatou sets, components of Julia sets.

Recommended books:

1. Iteration of rational functions, A. F. Beardon, Springer.
2. Complex dynamics, L. Carleson and T. W. Gamelin, , Springer.
3. Holomorphic dynamics, S. Morosawa, Y. Nishimura, M. taniguchi, T. Ueda, Cambridge University Press.
4. Dynamics of transcendental functions, X. H. Hua,C. C. Yang, Gordan and Breach Science Pub.
5. Dynamics in one complex Variable, John Milnor, Annals of Mathematics Studies, Princeton University Press.

MMT-E308 – Lattices and Ordered Structures

Unit-I

Partial orders and equivalence relations. Dual order, total order, lexicographic order. Chains and anti-chains, Zorn's lemma. Well-Ordering Principle and its equivalent forms. Principle of Duality, top and bottom elements, Hasse diagram. Order-preserving mappings principal down and up sets. Residuated mappings and related results.

Unit-II

Closures: dual closure, fixed points, bicomplete ordered sets. Isomorphisms of ordered sets: order isomorphism, dually isomorphic ordered sets. Galois connections: left and right annihilators. Semigroups of residuated mappings: Generalised Baer semigroup and related results.

Unit-III

Semilattices and lattices, lower and upper bounds of lattices. Down-set lattices, maximal and minimal elements morphisms. Sublattices: ideals, filters and related results. Lattice morphisms and related results.

Unit-IV

Complete lattices: definition, examples and related results. Knaster theorem and Bernstein theorem. Embedding of an ordered set: Dedekind–MacNeille Theorem. Baer semigroups: Definition and related results. Complemented lattices: examples and related results. Boolean algebras and Boolean rings.

Recommended books:

1. Lattices and Ordered Algebraic Structures, T.S. Blyth, Springer.
2. Lattice Theory, G. Birkhoff, American Mathematical Society.
3. Sets, lattices and Boolean Algebras, J.C. Abbott, Allyn & Bacon, Boston.

MMT-E309 – Topics in Ring Theory

Unit-I

Definitions and simple properties, embedding in a ring with unity, division rings and fields. Ideals and homomorphism, prime ideals in commutative rigs. Algebras, Power series rings, Direct sums, sums and direct sums of ideals, ideal products and Nilpotent ideals, some conditions on ideals and ideals in complete matrix rings.

Unit-II

Subdirect sums of rings and sub directly irreducible rings. Boolean rings and their properties. Prime ideals and prime radical, Semiprime ideals. The prime radical of a ring, prime rings, the descending chain conditions and the prime radical.

Unit-III

Ring of endomorphisms, the irreducible rings of endomorphisms, R-modules and rings of endomorphisms, irreducible rings and vector spaces. Dense rings of linear transformations, subspaces and descending chain conditions, some properties of the ring D_n. The Wedderburn-Artin Theorem.

Unit-IV

The Jacobson radical; preliminary concepts, definition and simple examples, further properties of the Jacobson radical, the descending chain condition, idempotents and regular elements of dense rings, some further results on dense rings, another radical of a ring.

Recommended books:

1. The Theory of Rings, Neal H. McCoy, Chelsea Publishing company.
2. Rings and ideals, Neal H. McCoy, The Carus Mathematical Monographs.
3. Noncommutative rings, I.N Herstien, The Carus Mathematical Monographs.
4. Introduction to Noncommutative Algebra, Matej Bresar, Springer.

MMT-E310 – Algebraic Topology

Unit-I

Paths and path homotopy with examples, homotopy equivalence, contractibility, deformation retracts and examples. Basic constructions: cones, mapping cones, mapping cylinders, suspension.

Unit-II

Fundamental groups: the fundamental group of the circle and its applications (including Fundamental Theorem of Algebra, Brouwer Fixed Point Theorem and Borsuk-Ulam Theorem). Van Kampen's Theorem, Covering spaces. Lifting of paths and homotopies, lifting properties.

Unit-III

Deck transformations and group actions, orbit spaces universal coverings (existence theorem optional), regular covering spaces and quotient spaces.

Unit-IV

Cell complexes, subcomplexes with examples. Simplicial complexes, barycentric subdivision, stars and links with examples. Simplicial Homology, Mayer-Vietoris Sequences, long exact sequence of pairs and triples.

Recommended books:

1. Algebraic Topology, A. Hatcher, Cambridge Univ. Press, Cambridge, 2002.
2. Algebraic topology: A First Course, W. Fulton, Springer-Verlag, 1995.
3. A Basic Course in Algebraic Topology, W. Massey, Springer-Verlag, Berlin, 1991.
4. An Introduction to Algebraic Topology, J. J. Rotman, Springer (India), 2004.
5. Elements of Algebraic Topology, J. R. Munkres, Addison-Wesley, 1984.

MMT-E311 – Topics in Applied Mathematics

Unit-I

Finite differences, Forward differences, Backward differences, Central differences, Symbolic relations and separation of symbols, detection of errors. Interpolation, Newton's formulae for interpolation, Lagrange's interpolation formula, its errors, Hermite's interpolation formula, Gauss forward and backward formulae. Numerical differentiation and its errors, Cubic spline method. Numerical integration, trapezoidal rule, Simpson's 1/3 and 3/8 rules. Evaluation of double integrals by trapezoidal and Simpson's rule.

Unit-II

Eigenvalue problem, eigenvalues of symmetric Tridiagonal matrix. Legendre polynomials, linearly independent functions, orthogonal functions. Numerical solution of ordinary differential equations, solution by Taylor's series, Picard's method, Euler's method, modified Euler's method, Runge-Kutta methods. Numerical solution of integral equations, method of degenerate Kernel's.

Unit-III

Periodic Functions. Piecewise Continuous Functions. Fourier Series and the need for Fourier Series. Dirichlet Conditions. Odd and Even Functions. Half-Range Fourier Sine and Cosine Series. Parseval's Identity. Uniform Convergence. Integration and Differentiation of Fourier Series.

Unit-IV

Fourier transform, inverse Fourier transform, Fourier sine and cosine transforms and their inversion, properties of Fourier transforms, Fourier transform of the derivative, convolution theorem, discrete Fourier transform and fast Fourier transform and their properties, applications of Fourier transform in partial differential equations with special reference to heat and wave equation.

Recommended books:

1. Numerical methods, M. K. Jain, S. R. K. Iyengar and R K Jain, New AGE; 6th edition.
2. Introductory Methods of Numerical Analysis, S.S. Sastary, Prentice Hall India Learning Private Limited; fifth edition.
3. Applied Numerical methods, C.F. Gerald and P.O. Wheatley, Pearson Addison Wesley, Greg Tobin; 7th edition.
4. Numerical solution of differential equations, M. K. Jain, New AGE International Publishers; fourth edition.
5. An Introduction to Laplace Transforms and Fourier Series, Phil Dyke, Springer Undergraduate Mathematics Series.
6. Fourier Analysis Theory And Problems, Murray R. Spiegel, Schaum's Outline Series.

M.A/M.Sc. Mathematics
Revised Syllabus

Semester-IV

Course Structure				**Marks**			
S.No.	**Course Code**	**Course Title**	**Credits**	**CIA**	**MT**	**ET**	**TOTAL**
1	MMT-C401	Abstract Measure Theory	4	20	30	50	100
2	MMT-C402	Field and Galois Theory	4	20	30	50	100
3	MMT-C403	Mathematical Statistics	4	20	30	50	100
Any Three of the Following							
4	MMT-E404*	Project Work	4	20†	30††	50†††	100
5	MMT-E405	Theory of Semigroups-II	4	20	30	50	100
6	MMT-E406	Category Theory	4	20	30	50	100
7	MMT-E407	Algebraic Geometry	4	20	30	50	100
8	MMT-E408	Advanced Topics in Graph Theory	4	20	30	50	100
9	MMT-E409	Analytic Theory of Polynomials	4	20	30	50	100
5	MMT-E410	Calculus of Variations and Integral Equations	4	20	30	50	100
10	MMT-E411	Differentiable Manifolds	4	20	30	50	100
11	MMT-E412	Functonal Analysis-II	4	20	30	50	100
12	MMT-E413	Special Functions	4	20	30	50	100
13	MMT-E414	Operation Research	4	20	30	50	100
12	MMT-E415	Algebraic Cryptography	4	20	30	50	100
13	MMT-E416	Algebraic Number Theory	4	20	30	50	100
12	MMT-E417	Wavelets and Frames	4	20	30	50	100
Total			24	120	180	300	600

∗ The eligibility of choosing the project as one of the electives is to qualify the interview/ test which is to be conducted by the department. In addition to this, the desirous student must have at least 60% marks with no backlogs in the previous three semesters.
† Presubmission Test, †† Dissertation(External), ††† Viva-voce (20 Internal & 30 External)

Semester IV

MMT-C401 – Abstract Measure Theory

Unit-I

Monotone functions and their properties, Vitali cover and Vitali covering lemma, differentiability of monotone functions. Bounded variation and absolute continuity and their relationship. Cantor set and Cantor Lebesgue function. Indefinite integral of Lebesgue integrable functions and its absolute continuity. Fundamental theorem of calculus

Unit-II

A brief introduction of L_p-spaces. Young's inequality, Hölders inequality, Minkowski's inequality, Cauchy-Schwarz's inequality and their applications. Completion of $C[a, b]$. Improper integral as Lebesgue integral, differentiation under the integral sign, calculation of some improper integrals.

Unit-III

Product measure, Iterated integrals, examples of non Lebesgue integrable functions, Fubbibi's theorem and Tonells's theorem. Collection of all measures on σ-algebra is a lattice, signed measure and Hahn decomposition theorem.

Unit-IV

Absolutely continuous signed measure and related results, singular measures, Radon-Nikodym theorem, Radon-Nikodym derivative, density function. Riesz-Fischer theorem, $L_1^\star(\mu) = L_\infty(\mu)$, $L_p(\mu)$ is a reflexive Banach space for $1 < p < \infty$.

Recommended books:

1. Real Analysis, H. L. Royden, P. M. Fitz Patrick, Pearson.
2. Real and Complex Analysis, Walter Rudin, McGraw Hill Education.
3. An Introduction to Measure and Integration, I. K. Rana, Narosa.

MMT-C402 – Field and Galois Theory

Unit-I

Field extensions: basic results and examples, degree of extension and related results, finite and algebraic extensions, finitely generated and simple extensions. Classical straightedge and compass constructions.

Unit-II

Splitting fields: examples and related results, existence and uniqueness of splitting fields, Algebraic closure and algebraically closed fields, separable and inseparable extensions, prefect fields. Characterization of finite fields, polynomials over finite fields, traces and norms. Cyclotomic polynomials and extensions.

Unit-III

Field automorphisms and fixed fields, Galois groups and Galois extensions. Fundamental Theorem of Galois Theory, with applications and related results. Composite extensions, cyclotomic extensions and abelian extensions over $\mathbb{Q}$.

Unit-IV

Symmetric and elementary symmetric functions. Discriminant of a polynomial, Galois groups of polynomials of degree 2, 3 and 4. Radical extensions, Solvability by radicals of cubic and quartic equations and Insolvability of the quintic.

Recommended books:

1. Abstract Algebra, D. S. Dumit and R. M. Foote, Wiley Sons.
2. Finite Fields, Lidl Rudolf, Cambridge University Press, 2nd Edition.
3. Field and Galois Theory, Morandi Patrick, Springer.
4. Galois Theory, Artin Emil, Dover Books on Mathematics.
5. Galois Theory, Ian Stewart, Chapman and Hall.

MMT-C403 – Mathematical Statistics

Unit-I

Discrete random variables and transformations, continuous random variables and transformations, expectation of random variables, some special expectations, Chebyshev's inequality, Markov's inequality. Distribution of two random variables, expectations, transformations.

Unit-II

Conditional distributions and expectations, the correlation coefficient, independent random variables, extension to several random variables, transformation for several random variables, linear combinations of random variables.

Unit-III

Some special distributions:: Binomial distribution, Bernoulli distribution, trinomial distribution, multinomial distribution, Negative binomial distribution, Poisson distribution Gamma distribution, chi-square distribution, beta distribution, Cauchy distribution, exponential, normal distribution, $t-$distribution and $F-$distribution. Student's theorem.

Unit-IV

Convergence in probability, convergence in distribution, weak law of large numbers, consistent estimator, sample variance as a consistent estimator of population variance, central limit theorem, maximum likelihood estimation,, Rao-Cramer inequality.

Recommended books:

1. Robert V. Hogg, Joseph McKean and Allen T. Craig, Introduction to Mathematical Statistics, Pearson Education Inc.

2. P. G. Hoel, S. C. Port and C. J. Stone, Introduction to probability, Universal Book Stall, New Delhi.

3. Sheldon Ross, A first course in probability, MacMillan Co. NY.

4. Vijay K. Rohatgi and A. K. Md. Ehsanes Saleh, An Introduction to Probability and Statistics, John Wiley & Sons.

MMT-E404 – Project Work

Presubmission Test

Presubmission test shall be based on the topic allotted by the concerned Guide/Supervisor. The pattern of the question paper and the marks distribution of the course shall be uniform across all groups.

Dissertation and viva voce

Project dissertation would consists of 04 chapters based on the survey of at least one research article published in SCI/SCOPUS indexed journal. The dissertation shall be evaluated by an external expert and has to be defended by the student in an open viva-voce. The external expert shall also be part of the committee for the conduct of viva-voce.

Recommended books:

- Books shall be recommended by the concerned Guide/Supervisor, who will conduct the examination of the course, accordingly.

MMT-E405 – Theory of Semigroups-II

Unit-I

Compeletely 0-simple semigroups, Regular Rees Matrix semigroups, Rees Theorem on completely 0-simple semigroups and related results. Isomorphism and normalization of completely 0-simple semigroups, congruences on completely 0-simple semigroups.

Unit-II

Completely regular semigroups, completely regular semigroups as semilattices of completely simple semigroups. Clifford semigroups as strong semilattices of groups, isomorphism of clifford semigroups. Review of inverse semigroups, the natural order relation on inverse semigroups. Brandt semigroups, definitions, examples and related results.

Unit-III

Congruences on inverse semigroups, definitions, examples and basic properties, kernel, trace congruence pairs, normal congruences on inverse semigroups, idempotent separating congruence, group congruences, maximum and minimum idempotent separating congruence.

Unit-IV

Varities of $(2, 1)$ algebras, equational classes and varieties, examples of some well known varieties of semigroups. Varieties of bands. Structure Theorems on some varieties of bands.

Recommended books:

1. Inverse Semigroups, Mario Petrich, John Wiley and Sons.
2. Fundamentals of Semigroup Theory, J. M. Howie, Oxford University Press, 1994.
3. Nine Chapters on the Semigroup Art, Alan J. Cain, Lecture Notes Porto, 2013.

MMT-E406 – Category Theory

Unit-I

Unit-I

Categories: definitions, examples and basic properties, isomorphisms. Construction on categories: opposite categories and duality principle, product of categories, arrow category, slice category, large, small and locally small categories. Functors: examples and basic properties. Covariant and contra-variant functors. Full, faithful, amnestic functors and embeddings. Equivalences and equivalent categories.

Unit-II

Subcategories, full subcategories examples and properties. Fully embeddable categories, isomorphism.Closed categories and skeleton, Monoidal categories. Reflections, co-reflections, reflective and co-reflective subcategories and related results. Natural transformations and natural isomorphisms. Representable functors and yoneda lemma.

Unit-III

Initial and terminal objects, zero objects. Separators and co-separators, sections and retractions. Monomorphisms and epimorphisms, bimorphisms and balanced categories. Equalizers and co-equalizers.

Unit-IV

Regular and extremal monomorphisms and epimorphisms. Pullbacks and pushouts. Products and coproducts. Limits and co-limits. Complete and co-complete categories.

Recommended books:

1. Abstract and concrete categories, J. Adamck, H. Herlich, and G. Strecuer, Dover Books on Mathematics.

2. Handbook of categorical Algebra-I, F. Borcuex, Cambridge University Press.

3. Category Theory, S. Awodey, Oxford University Press.

MMT-E407 – Algebraic Geometry

Unit-I

The Zariski topology and affine Space, going back and forth between subsets and ideals, irreducibility in the Zariski topology, irreducible closed subsets correspond to ideals whose radicals are prime, Zariski topology on the affine line.

Unit-II

The Noetherian property in topology and in algebra, Noetherian decomposition of affine algebraic subsets into affine varieties, topological dimension, Krull dimension and heights of prime ideals, ring of polynomial functions on an affine variety.

Unit-III

Geometric hypersurfaces and algebraic hypersurfaces, affine coordinate rings of functions on affine varieties, affine variety and the maximal spectrum, open sets and basic open sets for the Zariski topology, ring of functions on a basic open set in the Zariski topology.

Unit-IV

Quasi-compactness in the Zariski topology; regularity of a function at a point of an affine variety, global regular function on a quasi-affine variety, morphisms between affine or quasi-affine varieties, morphisms and affines as k-Algebra maps.

Recommended books:

1. Algebraic Geometry, Robin Hartshorne, Graduate Texts in Mathematics GTM52, Springer.
2. The Red Book of Varieties and Schemes, David Mumford, Springer.
3. Algebraic Geometry: a first Course, Joe Harris, Springer-Verlag, New York (1992).
4. An Introduction to Commutative Algebra, M. F. Atiyah and I. G. Macdonald,Addison-Wesley.
5. Algebraic Curves: an Introduction to algebraic geometry, William Fulton, London (1969).

MMT-E408 – Advanced Topics in Graph Theory

Unit-I: Coloring of Graphs

Vertex coloring, Brook's theorem, Nordhaus-Gaddum Theorem, Edge colouring of graphs, Konig's theorem, Vizing's theorem, Region coloring graphs, five color theorem, Heawood map colouring theorem, Tait's theorem, uniquely colourable graphs, chromatic polynomial of complete graph, totally disconnected graph, cycle, tree. Chromatic polynomial by identification of vertices, Hassler Whitney's Theorem.

Unit-II: Matchings and Factorization

Matchings, Berge's theorem, Hall's condition, The marriage theorem, perfect matchings, Factors, Tutte's 1−factor theorem, perfect matchings in regular bipartite graphs, Peterson's theorem, factorisation of complete graphs.

Unit-III: Edge Graphs and Digraphs

Edge graph, Krausz's theorem, Beineke's theorem, edge graphs of trees and Eulerian property of edge graphs. Digraphs, strong, weak and unilateral digraphs, Euler digraphs, Matrices of digraphs: Incidence matrix, cycle matrix, Cut-set matrix, adjacency matrix, Tournaments, Redei's theorem, Camion's theorems.

Unit-IV: Algebraic Graph Theory: An Introduction

Automorphism groups of graphs, Cayley group of a graph, Spectrum of some graphs − complete graph, complete bipartite, cycle and path, regular graph, compliment of a graph, edge graph. Determinant of adjacency matrix and Sach's theorem, Laplacian matrix, rank of Laplacian matrix, Laplacian spectrum of regular graphs, relation between the Laplacian spectrum of graph and its complement.

Recommended books:

1. Chromatic Graph Theory, Gary Chartrand, Ping Zank, CRC Press Taylor and Francis group.
2. An Introduction to Graph Theory, S. Pirzada, Universities Press, Orient Blackswan, Hyderabad, India, (2012).
3. A Text Book of Graph Theory, R. Balakrishnan, Ranganathan, Springer- Verlag.
4. Extremal Graph Theory, B. Bollobas, Academic Press.
5. A First Book of Graph Theory, J. Clark and D. A Holton, World Scientific.

MMT-E409 – Analytic Theory of Polynomials

Unit-I

The fundamental theorem of algebra (revisited). Symmetric polynomials. The Continuity theorem. m-Distribution and the associated system of orthogonal polynomials: general properties and three-point recurrence formula. Christoffel-Darboux and Gaussian quadrature formulae. Trigonometric and classical orthogonal polynomials. Tools from Matrix Analysis: Companion matrix, HardmardÂ's theorem, Greschgorin discs and Greschgorin disc Theorem.

Unit-II

Critical points of a polynomial and their relationship with the zeros: critical points as convex linear combination of zeros of a polynomial. Convex Hulls and Gauss-Lucas theorem. An Extension of Gauss-Lucas theorem: KuiperÂ's Theorem. Average distance from a line or a point. Real polynomials and JensonÂ's theorem. Extension of JensonÂ's theorem.

Unit-III

Derivative estimates on the unit disc: BernsteinÃ,Â's inequality, some of its generalizations and refinements due to Rahman and Schmeisser and A. Aziz. Derivative estimates for polynomials with restricted zeros: inequalities due to ErdösÂ–Lax and Paul Turán. Derivative estimates on the unit interval: BernsteinÂ's pointwise estimate and inequality due to A. Markov. Extensions to higher order derivatives. Result due to Schur.

Unit-IV

Rational functions and some inequalities for the maximum modulus of the derivative of a rational function with prescribed poles: extension of inequalities due to Bernstein, ErdösÂ–Lax and Paul Turán. Derivative estimate for functions of exponential type on the real line: analogue of BernsteinÂ's inequality for functions of exponential type. L_p analogues of some polynomial inequalities: inequalities due to Zygmund and De Bruijn.

Recommended books:

1. Analytic theory of Polynomials, Q.I. Rahman and G. Schmeisser, Clarendon Press.
2. Geometry of polynomials, Morris Marden, American Mathematical Society.
3. Topics in polynomials: extremal properties, problems, inequalities, zeros, G.V. Milovanovic, D.S. Mitrinovic and Th. M. Rassias, World Scientific.
4. Problems and theorems in Analysis II, G. Polya and G. Szego, Springer

Suggested Papers:

1. Bernstein-Type Inequalities for Rational Functions with Prescribed Poles, Xin Li, R. N. Mohapatra And R. S. Rodriguez, J. London Math. Soc. (2) 51 (1995) 523-531.

MMT-E410 – Calculus of Variations and Integral Equations

Unit-I

Calculus of Variations: Introduction, differentiability and the first Variation of a Functional. Necessary Condition for Extrema. The Fundamental Lemma of the Calculus of Variations and the Euler-Lagrange Equation. Some Typical Problems of The Subject-The Brachistochrone Problem. Lagrange Multipliers and Isoperimetric Problems.

Unit-II

Variational Problems with Moving (or Free) Boundaries. The Second Variation and Sufficient Condition for Extremum of a Functional. Approximate Solutions of Boundary Value Problems by Rayleigh-Ritz Method.

Unit-III

Linear Integral Equations: Introduction, Linear Integral Equation of The First and Second Kind of Fredholm And Volterra Type. Solution of Fredholm Integral Equations of The Second Kind with Separable Kernels. Solution of Volterra Integral Equations of First and Second Kind.

Unit-IV

Orthonormal System of Functions, Eigen Values and Eigen Functions. Hilbert-Schmidt Method of Solving Non-Homogeneous Fredholm Integral Equations of Second Kind.

Recommended books:

1. Calculus of variation, M. Gelfand and S. V. Fomin, Dover.
2. Calculus of variations with applications to physics, R. Weinstocks, Dover.
3. Integral equations, F. G. Tricomi, Dover.
4. Introduction to nonlinear differential equations and integral equations, H. L. Davis, Dover.

MMT-E411 – Differentiable Manifolds

Unit-I

Calculus of $\mathbb{R}^n$: Continuity and differentiability of functions from $\mathbb{R}^n \to \mathbb{R}^n$, Chain rule, Differential map, Inverse mapping theorem and its implications in geometry, diffeomorphisms.

Unit-II

Manifolds: Topological manifold, Atlas, smooth Manifold, Examples of manifolds, Differentiable structure on a manifold, Space of smooth maps, Differential of a smooth map

Unit-III

Submanifolds: Immersion, Embedding and submanifolds with examples, tangent space of submanifolds, Vector fields and smooth maps, Lie brackets, Vector bundles, Cotangent bundle, tangent covectors on manfiolds.

Unit-IV

Differential forms and connections: Tensors, Contravariant and covariant tensors, Mixed tensors, Alternating tensors, Differential forms, Pull back, Differential forms on manifolds, Exterior derivative.

Recommended books:

1. A course in Differential Geometry and Lie groups, S. Kumaresan, Hindustan Book Agency.

2. An introduction to Differential Manifolds and Riemannian Geometry, W. M. Boothby, Academic Press; 2nd edition.

3. J. M. Lee: Introduction to Smooth Manifolds, Graduate Texts in Mathematics, Springer.

4. M. Spivak: A comprehensive introduction to differential geometry, Publish or Perish, Inc., Houston, Texas, 2005.

MMT-E412 – Functional Analysis-II

Unit-I

Weak and weak* topologies on a Banach space, Dual of a Banach space in its weak* topology, Goldstine's theorem, Banach-Alaoglu theorem and its simple consequences, Reflexivity of Banach spaces and weak compactness, Geometric and separation forms of Hahn-Banach Theorem, Applications of Hahn-Banach Theorem to: Banach limits, Markov-Kakutani theorem for a commuting family of maps, Dual of Subspace, Quotient space of a normed space.

Unit-II

Complemented subspaces of Banach spaces, Dixmier's theorem on the complement- tability of the dual of a Banach space in its bidual, uncomplementability of C_0 in ℓ_∞ and its consequences, Banachs closed range theorem, characterizations of injective and surjective bounded linear mappings between Banach spaces.

Unit-III

Completeness of $L_p[a, b]$, Duals of ℓ_∞ and L_p spaces, Mazur-Ulam theorem on isometries between real normed spaces, applications of fundamental theorems of functional analysis to Radon-Nikodym Theorem, non-surjectivity of Laplace transform and Muntz theorem for $C[0, 1]$.

Unit-IV

ℓ_∞ and $C[0, 1]$ as universal separable Banach spaces, ℓ_1 as a quotient universal separable Banach space. Extreme points, Krein-Milman theorem and its simple consequences, Banach Stone Theorem on isometries between $C(X)$ and $C(Y)$.

Recommended books:

1. Functional Analysis, B. V. Limaye, New Age International Publishers.
2. Infinite Dimensional Analysis, C. D. Aliprantis and K C Border, Springer.
3. A first course in functional Analysis, C. Goffman and G. Pedrick, Prentice Hall.
4. A Course in Functional Analysis, J.B. Conway, Springer Verlag.
5. Introduction to Banach Spaces and their geometry, B. Beauzamy, North Holland, Elsevier Science Ltd.
6. Functional Analysis, W. Rudin, Tata McGraw Hill.
7. An Introduction to Banach Space Theory, R. E. Megginson, Springer Verlag.
8. Linear Analysis, Ballobas. B, Camb. Univ. Press.

MMT-E413 – Special Functions

Unit-I

Review of Infinite products: definition, necessary condition for convergence, associated series of logarithms, absolute and uniform convergence of infinite products. The Euler or Mascheroni constant γ and the WeierstrassÃ,Â's definition of the Gamma function Γ. A Series for $\frac{\Gamma'(z)}{\Gamma(z)}$. EulerÃ,Â's integral for $\Gamma(z)$. The Beta function. The Factorial function. Legendre's duplication formula. Gauss's multiplication theorem. The behaviour of $\log\Gamma(z)$ for large $|z|$.

Unit-II

The Hypergeometric function: a simple integral representation of the function $F(a, b, c; z)$. $F(a, b, c; 1)$ as a function of the parameters. Evaluation of $F(a, b, c; 1)$. The contiguous function relations. The hypergeometric differential equation, logarithmic solutions of the hypergeometric equation. $F(a, b, c; z)$ as a function of its parameters. Elementary series manipulations. Simple transformations. Relation between functions of z and $1 - z$. Quadratic transformations, theorem due to Kummer.

Unit-III

Orthogonal Polynomials: Simple sets of polynomials, orthogonality, an equivalent condition for orthogonality. Zeros of orthogonal polynomials. Expansion of polynomials. The three-term recurrence relation. The Christoffel-Darboux formula. Normalization, Bessel's inequality. Legendre, Hermite, Laguerre And Jacobi Polynomials.

Unit-IV

Generating Functions: The generating function concept. *Bessel Functions:* definition of $J_n(z)$, Bessel's differential equation, Bessel's integral. Modified Bessel functions. *Elliptic Functions:* doubly periodic functions, elementary properties, order of an elliptic function. The Weierstrass function $\mathcal{P}(z)$. Other elliptic functions. A differential equation for $\mathcal{P}(z)$. Connection with elliptic integrals.

Recommended books:

1. Special Functions, Earl D. Rainville, The Macmillan company - New York.

2. Special Functions & Their Applications, N. N. Lebedev, Dover Books on Mathematics.

3. Special Functions and Orthogonal Polynomials, Refaat El Attar, Lulu.com.

MMT-E414 – Operations Research

Unit-I

Introduction to Operation Research, Linear Programming Problem: mathematical formulation for LP problems and Graphical method. Definitions: Slack and Surplus variables, matrix form of LPP, feasible solution, basic solution, basic feasible solution, degenerate and non-degenerate solution. Redundant constraint, reduction of a feasible solution to a basic feasible solution. Convex sets: the set of all feasible solutions to an LPP is a convex set, Extreme point theorem, Simplex method, Flowchart for simplex method.

Unit-II

Two Phase method, Big-M method for solving LP problems with optimal solution, infeasible solution, degenerate solution, unbounded solution and multiple optima solution, Concept of duality in Linear programming, formation of Primal-Dual problems, dual of a dual is primal. Weak duality theorem, Strong duality theorem, examples.

Unit-III

Transportation Problems: Mathematical formulation, necessary and sufficient condition for the existence of feasible solution, loops in TP problems. Methods for initial basic feasible solution: North-West corner method, Least cost method, Vogel's approximation method, U.V method. Assignment problems: Hungarian method, Sensitivity Analysis: changes in the coefficients of the objective function, adding and deletion of a constraint and a variable.

Unit-IV

Project Management: construction of network, Construction by critical path method (CPM) and by PERT (probability consideration in PERT). Game theory: Two person zero sum games, games with pure strategies, mixed strategies, Min. Max. principle, dominance rule, solution of $2 \times 2, 2 \times n, m \times 2$ games.

Recommended books:

1. Operations Research: An Introduction, Hamdy Taha, Pearson.
2. Operations Research, Kanti Swarup, P. K. Gupta and Man Mohan, Sultan Chand and Sons.
3. Operations Research, Richard Bronson and Govindasami Naadimuthu, Schaum's Outline Series, Tata McGraw Hill.

MMT-E415 – Algebraic Cryptography

Unit-I

Introduction to modern cryptography and its principles, data encryption and data decryption, Symmetric (private) and Asymmetric (public) cryptography, Some well known algorithms: AES, DES, IDES, and Random number generator (RNG, Classical cryptography, simpe substitution cipher, vigenere cipher,caesar cipher, Hill cipher and affine Hill cipher, cryptanalysis and brute force attack.

Unit-II

Introduction to asymmetric cipher, origin of public key cryptography, the discrete logarithm problem, Diffie-Hellman key exchange algorithm, the Elgamal public key cryptosystem, hardness of discrete logarithm problem.

Unit-III

Integer factorization and RSA cryptosystem, implementation and security issues of RSA cryptosystem, Euler's and Fermat's formula, primality testing, Miller-Rabin test for composite numbers, introduction and implementations of elliptic curve cryptography, hash function and its applications.

Unit-IV

Digital signatures: components of a digital signature scheme, RSA digital signature, Elgamal digital signature, digital signature algorithm (DSA),Goldreich-Goldwasser-Halevi (GGH) lattice based digital signature scheme, NTRU digital signature scheme, stream cipher vs block cipher, one time pad, linear feedback shift register.

Recommended books:

1. An introduction to mathematical cryptography, J. Hoffstein, J. Pipher, J. Silverman, Springer.
2. Cryptography and network security, William stalling, Pearson.
3. Understanding cryptography, C. paar, J. Pelzl, Springer.

MMT-E416 – Algebraic Number Theory

Unit-I

Rings of integers, Dedekind Domains, Unique factorization of ideals, the ideal class group, Discrete valuations, integral closure of Dedekind Domains, factorization in extensions, examples.

Unit-II

Algebraic number fields and their conjugates, Algebraic integers, norm, Different and Discriminant of a number field, Integral Basis of an algebraic number field, multiplication and divisibility of ideals.

Unit-III

Norm of an integral domain, Norm and trace of an element, Norm of product of ideals and fractional ideals, Norm of a prime ideal, embedding, Dirichlet's Unit theorem, Fundamental system of units, S-units.

Unit-IV

Factoring primes in quadratic field, Monogenic number field, cubic field,arbitrary number field, Cyclotomic field, units in real quadratic fields, units of norm, 1 & -1, Absolute values, Non-Archimedean absolute values, equivalent absolute values.

Recommended books:

1. Introductory Algebraic Number Theory, Saban Alaca and Kenneth S. Williams, Cambridge.
2. Algebraic Number Theory, J. S. Milne, Springer.
3. Algebraic Number Theory, Robert B. Ash, Dover Publications.
4. Algebraic Number Theory, Serge Lang, Springer.

MMT-E417 – Wavelets and Frames

Unit-I

Definition and examples of Fourier Transform in $L^2(\mathbb{R})$, basic properties of Fourier transforms, Plancherel's and Parseva'ls formlae, energy preserving relation, Convolution and Correlation, Poisson summation formula, Shannon-Whittaker, sampling theorem, Heisenberg's uncertainty principle.

Unit-II

Motivation and definition of continuous wavelet transforms (CWT), basic properties of wavelet transforms, Haar wavelet, Maxican hat wavelet, Meyer wavelet and their Fourier transforms, Parseval's, formula and energy preserving relation of CWT, reconstruction formula for CWT. Discrete wavelet transform and applications.

Unit-III

Finite frames, Canonical reconstruction formula, Frames and matrices, Similarity and unitary equivalence of frames, Frame bounds and frame algorithms.

Unit-IV

Frames and Bessel sequences in infinite dimensional Hilbert spaces, Frame sequence, Gram matrix, Frames and operators, Characterization of frames, Dual frames, Tight frames, Continuous frames, Frames and signal processing, Tight frames and dual frame pairs.

Recommended books:

1. I. Daubechies, Ten Lectures on Wavelets, SIAM, Philadelphia, 1992.
2. D.K Ruch and P.J. Van Fleet, Wavelet Theory, John Wiley, 2009.
3. E. Hernandez and G. Weiss, A First Course on Wavelets, CRC Press, New York, 1996.
4. D. Han, K. Kornelson, D. Larson and E. Weber, Frames for Undergraduates, American Mathematical Society, Student Mathematical Library, Volume 40, 2007.
5. O. Christensen, An Introduction to Frames and Riesz Bases, Second Edition, Birkhäuser.