Course Structure and Syllabus for Integrated BSc–MSc Mathematics

YEAR: 2020 (BATCH: 2019) ONWARDS

Department of Mathematics
Central University of Kashmir
Ganderbal

Contents

Integrated BSc – MSc Mathematics
Revised Syllabus

Semester-I

Course Structure				Marks			
S.No.	**Course Code**	**Course Title**	**Credits**	**CIA**	**MT**	**ET**	**TOTAL**
1	MTH-101	Differential Calculus	4	20	30	50	100
2	MTH-102	Integral Calculus	4	20	30	50	100
3	PHY-101T	Physics-I	4	20	30	50	100
4	PHY-101L	Physics-I Lab	2	10	15	25	50
5	CHM-101T	Chemistry-I	4	20	30	50	100
6	CHM-101L	Chemistry-I Lab	2	10	15	25	50
7	GEC-101	Communication Skills	4	20	30	50	100
Total			24	120	180	300	600

Semester I

MTH-101 – Differential Calculus

Unit-I

Introduction to limits: $\epsilon - \delta$ approach. Algebra of limits. Limits at infinity. $\epsilon - \delta$ definition of continuity of a function. Algebra of continuous functions. Discontinuity and types of discontinuities. Differentiability of real valued functions of a real variable. Differentiation of parametric functions. Mean value theorems: Rolle's theorem, Lagrange's mean value theorem and Cauchy's mean value theorem.

Unit-II

Partial differentiation: definition and geometric interpretation. Total differential. Differentiation of composite and implicit functions. Homogeneous functions and Euler's theorem on homogeneous functions. Successive differentiation: calculation of nth derivatives and Leibnitz's theorem. Formal expansion of functions: Taylor's and Maclaurin's Theorem. Maxima and minima of functions. Indeterminate forms.

Unit-III

Tangents and normals: Equation of tangent and normal at any point of a smooth curve (polar form). Length of tangent, normal, sub-tangent and sub-normal at any point of a curve. Pedal equation. Arc length differential. Curvature: definition, radius of curvature and center of curvature. Evolutes and involutes.

Unit-IV

Envelopes: definition and determination of an envelope. Asymptotes: definition and determination of asymptotes. Multiple points: cusps, nodes and conjugate points; Conditions for a point to be a multiple point. Curve Tracing.

Recommended books:

1. Differential Calculus, Auzeem, Chopra and Kochhar, Kapoor Sons Srinagar.
2. Differential Calculus, Shanti Narayan, P. K. Mittal, S. Chand Publishing.
3. Calculus, James Stewart, Cengage Learning.

MTH-102 – Integral Calculus

Unit-I

Integration of irrational functions, Reduction formulae, Theorems on Definite integrals and their applications, Summation of series with the help of definite integrals. Beta and Gamma integrals and related results

Unit-II

Quadrature, Area of a region bounded by a curve, Sectorial Areas bounded by a closed curve, length of plane curves, volume and surface area of surfaces of revolution, Pappus theorem.

Unit-III

Ordinary differential equations: Graphical representation. Exact Equations, integrating factors and method of solution. First order linear differential equations, equations reducible to the linear form and their solution. Solution of homogeneous equations of the form $\frac{dy}{dx} = f(\frac{y}{x})$ and equations reducible to this form.

Unit-IV

Higher order linear differential equations. Linear differential equations with constant coefficients. Auxiliary equation and complementary function. Different cases for the complementary function. Particular integrals. Euler-Cauchy equations and their solutions. Legendre's linear differential equations and their solutions.

Recommended books:

1. Thoms Calculus, Maurice D. Weir, Joel Hass, Pearson.
2. Ordinary and Partial Differential Equations , H T H Piaggo, Narosa.
3. Integral calculus by Kochhar Chopra, Kapoor Publications.
4. Mathematical Methods for Scientists and Engineers, Donald A McQuarrie.

PHY-101T – Physics-I

Unit-I

Fundamentals of Dynamics: Reference frames. Inertial frames, Review of Newton's Laws of Motion. Dynamics of a system of particles. Centre of Mass. Principle of conservation of momentum. Impulse. Work and Energy: Work–Energy Theorem. Conservative and non- conservative forces. Potential Energy. Energy diagram. Stable and unstable equilibrium. Force as gradient of potential energy. Work done by non-conservative forces. Law of conservation of Energy.

Unit-II

Rotational Dynamics: Angular momentum of a particle and system of particles. Torque. Principle of conservation of angular momentum. Moment of Inertia. Calculation of moment of inertia for circular and spherical bodies. Kinetic energy of rotation. Motion involving both translation and rotation. Elasticity: Elastic potential energy stretched in a wire, Torsional pendulum

Unit-III

Gravitation and Central Force Motion: Law of gravitation. Gravitational potential energy. Inertial and gravitational mass. Potential and field due to spherical shell and solid sphere. Motion of a particle under a central force field (motion is in a plane, angular momentum is conserved, areal velocity is constant). Two-body problem and its reduction to one-body problem and its solution. Differential equation of orbit in central force field. Kepler's Laws. Satellite in circular orbit and applications.

Unit-IV

Special Theory of Relativity: Michelson-Morley Experiment and its outcome. Postulates of Special Theory of Relativity. Constancy of speed of light. Lorentz Transformations. Simultaneity of events. Length contraction. Time dilation. Twin Paradox. Relativistic addition of velocities. Relativistic Doppler Effect.

Recommended books:

1. An introduction to mechanics, D. Kleppner, R. J. Kolenkow, McGraw-Hill.
2. Mechanics, Berkeley Physics, Vol.I, C. Kittel, W. Knight, et.al.,Tata McGraw-Hill.
3. Feynman Lectures, Vol. I, R. P. Feynman, R. B. Leighton, M. Sands, Pearson Education.
4. Introduction to Special Relativity, R. Resnick, John Wiley and Sons.
5. Mechanics, D.S. Mathur, S. Chand and Company Limited.
6. University Physics, F. W Sears, M.W Zemansky, H. D Young , Addison Wesle.

PHY-101L – Physics-I Lab

1. To determine the Moment of Inertia of a Flywheel about its own axis of rotation
2. To determine the Young's Modulus of the given material.
3. To determine the Modulus of Rigidity of a Wire by Maxwell's needle.
4. To determine the Elastic Constants of a Wire by Searle's method.
5. To determine g by Bar Pendulum.

Recommended books:

1. Advanced Practical Physics for students, B.L. Flint and H.T. Workshop, Asia Publishing House.
2. Advanced level Physics Practicals, Michael Nelson and Jon M. Ogborn, Heinemann Educational Publishers
3. A Text Book of Practical Physics, Indu Prakashand Ramakrishna, Kitab Mahal, New Delhi.

CHM-101T – Chemistry-I

Unit-I

Theories of acids and bases. Relation between initial acid concentration, pKa and pH, Henderson-Hasselbalch equation, dependence of ionization on pH of solution, uses of the HH equation, titration of strong and weak acids with strong base. Exact treatment of the ionization of diprotic acid. Exact treatment of Bronsted lowery type monobase. Salt hydrolysis. Buffer mixtures (buffering range, buffering capacity). pH indicators.

Unit-II

Bohr's theory and its limitations, dual behaviour of matter and radiation, de-Broglie's relation, Heisenberg Uncertainty principle. Hydrogen atom spectra. Need for the quantum mechanical approach to atomic structure. Time independent Schrodinger equation and meaning of various terms in it. Significance of quantum numbers, Stability of half-filled and completely filled orbitals, concept of exchange energy. Relative energies of atomic orbitals. Anomalous electronic configurations. Periodicity in s- and p-block elements with respect to electronic configuration, atomic and ionic size, ionization enthalpy, electronegativity scales. Concept of unusual oxidation states, inert pair effect, diagonal relationship. Effective nuclear charge and its calculation using Slater rules.

Unit-III

Type of solution, Concentration terms, Roult's law, activity and activity coefficient Colligative properties (elevation of boiling point; depression of freezing point, osmotic pressure) van't Hoff theory of dilution, determination of molecular weight. Solubility of salts, Thermodynamic and apparent solubility products, salt or electrolyte effect, common ion effect.

Unit-IV

Electron displacements: Inductive Effect, Electromeric Effect, Conjugation and Hyperconjugation. Aromaticity: Criteria of aromaticity, Huckel's rule, Molecular orbital description of aromatic, non-aromatic and anti-aromatic systems. Aromaticity of benzenoid and non-benzenoid systems. Conformational Isomerism: Conformational analysis of ethane, butane and cyclohexane. Interconversion of Wedge, Newman, Sawhorse and Fischer projections. Configurational Isomerism: Geometrical isomerism, E-Z system nomenclature. Chirality, isomerism in systems with more than one chiral centres. Enantiomerism, Diastereomerism and Meso compounds. Threo and Erythro, D and L, R and S systems of nomenclature.

Recommended books:

1. A New Concise Inorganic Chemistry, E.L.B.S. by J. D. Lee.
2. Basic Inorganic Chemistry, John Wiley, by F.A. Cotton & G. Wilkinson.
3. Inorganic Chemistry, Oxford University Press by D. F. Shriver and P. W. Atkins.

CHM-101L – Chemistry-I Lab

(A) Volumetric Analysis.

1. Estimation of sodium carbonate and sodium hydrogen carbonate present in a mixture.
2. Estimation of oxalic acid by titrating it with $KMnO_4$.
3. Estimation of water of crystallization in Mohr's salt by titrating with $KMnO_4$.
4. Estimation of Fe (II) ions by titrating it with K_2Cr2O_7 using internal indicator.
5. Estimation of Cu (II) ions iodometrically using $Na_2S_2O_3$

(B) Qualitative Analysis Semi-micro qualitative analysis using H_2S of mixtures- not more than four ionic species (two anions and two cations and excluding insoluble salts) out of the following: Cations : $NH^+, Pb^{2+}, Ag^+, Bi^{3+}, Cu^{2+}, Cd^{2+}, Sn^{2+}, Fe^{3+}$, $Al^{3+}, Co^{2+}, Cr^{3+}, Ni^{2+}, Mn^{2+}, Zn^{2+}, Ba^{2+}, Sr^{2+}, Ca^{2+}, K^+$
Anions : $CO^{2-}, S^{2-}, SO^{2-}, SO^{2-}, NO^{3-}, CH_3COO^-, Cl^-, Br^-, I^-, NO^{3-}, SO^{42-}$, $PO^{43-}, BO^{33-}, C_2O^{42-}, F^-$ (Spot tests should be carried out wherever feasible) Any other practical found feasible by the teacher concerned.

Recommended books:

1. A.I. Vogel, Qualitative Inorganic Analysis, PrenticeHall,
2. A.I. Vogel, Quantitative Chemical Analysis, PrenticeHall,
3. Advanced Practical Inorganic Chemistry; Gurdeep Raj; Goel PublishingHouse;

GEC-101 – Communication Skills

Unit-I

Business Communication: Telephonic skill (oral communication and listening practice). Interview, specifiuc activities in writing skills, writing minutes, circulars, writing applications for jobs, CV writing.

Unit-II

Usage of English Language: Vocabolary development, word structure and phrasal verbs.

Unit-III

Comprehension: Listening and Reading comprehension, written communication and composition, paragraph writing, precis writng, dicto compostion, letter writing, , email writing, writing books reviews.

Unit-IV

Essentials of Grammar: Remidial exercises in parts of speech, structure of sentence, sequences of tenses, uses of articles ad modes of reporting, making short speeches, interprting visual presentations, role play and group discussions.

Recommended books:

1. Communicating a social and career focus, K. M. Berko, Andrew D. Welwyn and Darlyn R. Welwyn, Houghton Mifflin Co.
2. The Craft of Scientific Writing (3rd Edition), Michael Alley.
3. Science and Technical Writing – A Manual of Style, Philip Reubens.
4. Writing Remedies – Practical Exercises for Technical Writing. E.H. Weiss.

Integrated BSc – MSc Mathematics
Revised Syllabus

Semester-II

Course Structure				Marks			
S.No.	**Course Code**	**Course Title**	**Credits**	**CIA**	**MT**	**ET**	**TOTAL**
1	MTH-201	Complex Trigonometry and Theory of Equations	4	20	30	50	100
2	MTH-202	Mechanics	4	20	30	50	100
3	PHY-201T	Physics-II	4	20	30	50	100
4	PHY-201L	Physics-II Lab	2	10	15	25	50
5	CHM-201T	Chemistry-II	4	20	30	50	100
6	CHM-201L	Chemistry-II Lab	2	10	15	25	50
7	GEC-201	Environmental Science	4	20	30	50	100
Total			24	120	180	300	600

Semester II

MTH-201 – Complex Trigonometry and Theory of Equations

Unit-I

Review of complex number system, triangle inequality, Geometric representation of complex numbers, De-Moivre's Theorem for rational index and its applications: n-th roots of unity, trigonometric functions.

Unit-II

Functions of complex variables, exponential, circular, hyperbolic, inverse hyperbolic and logarithmic functions of a complex variable. Summation of trigonometric series, difference method and $C + iS$ method.

Unit-III

Relation between roots and coefficients of a polynomial equation, synthetic division, diminishing the roots of an equation by a given number, removal of terms of an equation, formation of equations whose roots are functions of the roots of a given equation, equation of squared differences. Cardon's method and Descarte's method.

Unit-IV

Symmetric functions, applications, symmetric functions of the roots. Transformation of equations. Solutions of reciprocal and binomial equations. Algebraic solutions of the cubic and bi-quadratic equations. Properties of the derived functions.

Recommended books:

1. W.S. Burnside and A.W. Panton, The Theory of Equations, Dublin University Press, 1954.
2. C. C. MacDuffee, Theory of Equations, John Wiley & Sons Inc., 1954.
3. Complex Trigonometry, M R Puri, Kapoor publications.

MTH-202 – Mechanics

Unit-I

Composition and resolution of forces acting at a point. Resultant forces, Parallelogram law of forces, Resolved parts of forces, $\lambda - \mu$ theorem and its applications. Triangle law of forces, Lami's Theorem and its converse parts with generalization, Equilibrium of forces

Unit-II

Parallel forces, Resultant of two parallel forces acting in a rigid body and its generalization. Moment of a force about a point, Coplanar forces acting on a rigid body, Moment of a force about a line, Varignor's Theorem, Couple, Moment of a couple, Equivalent couples, Composition of couple's, Resultant of a single force and a couple.

Unit-III

Motion of a particle, Velocity and Acceleration of a moving particle, Rectangular motion under uniform acceleration, Recti-linear motion, Relation between u, v, f, s and t. Vertical motion under gravity. Newtons laws of motion, Matter Mass momentum, the relation $p = mv$ Simple applications. Motion of a mas on a horizontal table and other hanging bodies.

Unit-IV

Simple Harmonic motion, Projectiles, Velocity and Acceleration of a particle moving in a curved path. Motion of particle, Path, Time of flight range etc. Velocity at any point, Direction to hit a given point.

Recommended books:

1. Statics by G. N. Hafiz and K.L. Gupta, Sultan Chand.
2. Elements of Dynamics K.L Gupta and K.C Pant, S Chand and Sons.
3. Dynamics S.R. Gupta, S Chand and Sons.
4. Statics and Dynamics, David M.C. Mahon, Publisher, McGraw-Hill Education.

PHY-201T – Physics-II

Unit-I

Vector Analysis: Vector Algebra (Overview), Vector Triple Products, How Vectors Transform Differential Calculus: Gradient, Divergence and Curl of vector fields, Product Rules, Second Derivatives Integral Calculus: Line, Surface, and Volume Integrals, Fundamental Theorem of Calculus, Fundamental Theorem for Gradients, Fundamental Theorem for Divergences and Fundamental Theorem for Curls Curvilinear Coordinates: Spherical and Cylindrical Coordinates, The Dirac Delta Function (1D and 3D).

Unit-II

Electrostatics: Introduction, Coulombs Law, Electric Field, Continuous Charge Distributions; Divergence and Curl of Electrostatic Fields: Electric Flux and Gauss's Law, Divergence of E, Application of Gauss Law. Electric Fields in Matter: Dielectrics, Induced Dipoles, Alignment of Polar Molecules, Polar molecules, Bound Charges and their Physical Interpretation, The Field inside a Dielectric. The Electric Displacement: Gauss's Law in the Presence of Dielectrics, Linear Dielectrics, Electric Susceptibility, Permittivity and Dielectric Constant.

Unit-III

Magnetostatics: The Lorentz Force Law: Magnetic Fields, Magnetic Forces: Cyclotron Motion, Cycloid Motion, Currents, Current density and Equation of Continuity. The Biot-Savart Law: Steady Currents, The Magnetic Field of a Steady Current, The Divergence and Curl of B, Ampere's Law and its Applications, Comparison of Electrostatics and Magnetostatics.

Unit-IV

Magnetic Fields in Matter: Magnetization, Effect of Magnetic Field on Atomic Orbitals Bound Currents and their Physical Interpretation. Ampere's Law: Ampere's Law in Magnetized Materials, Magnetic Susceptibility and Permeability, Ferromagnetism and Hysteresis Curve.

Recommended books:

1. Electricity, Magnetism & Electromagnetic Theory, S. Mahajan and Choudhury, Tata McGraw-Hill
2. Electricity and Magnetism, Edward M. Purcell, McGraw-Hill Education
3. Feynman Lectures Vol.2, R.P.Feynman, R.B.Leighton, M. Sands, Pearson Education
4. Elements of Electromagnetics, M.N.O. Sadiku, Oxford University Press.
5. Electricity and Magnetism, J.H.Fewkes & J.Yarwood. Vol. I, Oxford Univ. Press.

PHY-201L – Physics-II Lab

1. Use a Multi-meter for measuring (a) Resistances, (b) AC and DC Voltages,(c) DC Current, (d) Capacitances, and (e) Checking electrical fuses.
2. To determine an unknown Low Resistance using Carey Foster's Bridge.
3. To determine self inductance of a coil by Anderson's bridge.
4. To study response curve of a Series LCR circuit and determine its (a) Resonant frequency, (b) Impedance at resonance, (c) Quality factor Q, and (d) Bandwidth.
5. Measurement of charge and current sensitivity and CDR of Ballistic Galvanometer

Recommended books:

1. Advanced Practical Physics for students, B.L.Flint and H.T.Workshop, Asia Publishing House.
2. A Text Book of Practical Physics, I.Prakash & Ramakrishna, Kitab Mahal
3. Advanced level Physics Practicals, Michael Nelson and Jon M. Ogborn, Heinemann Educational Publishers
4. A Laboratory Manual of Physics for undergraduate classes, D. P. Khandelwal, VaniPub

CHM-201T – Chemistry-II

Unit-I

Aliphatic Hydrocarbons: Functional group approach for the following reactions (preparations & reactions) to bestudied in context to their structure. Alkanes: (Upto 5 Carbons). Preparation: Catalytic hydrogenation, Wurtz reaction, Kolbe's synthesis, from Grignard reagent. Reactions: Free radical Substitution: Halogenation. Alkenes: (Upto 5 Carbons) Preparation: Elimination reactions: Dehydration of alkenes and dehydrohalogenation of alkyl halides (Saytzeff's rule); cis alkenes (Partial catalytic hydrogenation) and trans alkenes (Birch reduction). Reactions: cis-addition (alk. $KMnO_4$) and trans-addition (bromine), Addition of HX (Markownikoff's and anti- Markownikoff's addition), Hydration, Ozonolysis, oxymecuration- demercuration, Hydroboration-oxidation. Alkynes: (Upto 5 Carbons) Preparation: Acetylene from CaC2 and conversion into higher alkynes; by dehalogenation of tetra halides and dehydrohalogenation of vicinaldihalides. Reactions: formation of metal acetylides, addition of bromine and alkaline $KMnO_4$, ozonolysis and oxidation with hot alk. $KMnO_4$.

Unit-II

Aromatic hydrocarbons:Preparation (Case benzene): from phenol, by decarboxylation, from acetylene, from benzene sulphonic acid. Reactions: (Case benzene): Electrophilic substitution: nitration, halogenation and sulphonation. Friedel- Craft's reaction (alkylation and acylation) (upto 4 carbons on benzene). Side chain oxidation of alkyl benzenes (upto 4 carbons on benzene). Alkyl Halides: (Upto 5 Carbons) Types of Nucleophilic Substitution ($SN1, SN2$ and SNi) reactions. Preparation: from alkenes and alcohols. Reactions: hydrolysis, nitrite & nitro formation, nitrile & isonitrile formation. Williamson's ether synthesis: Elimination vs substitution. Aryl Halides Preparation: (Chloro, bromo and iodo-benzene case): from phenol, Sandmeyer & Gattermann reactions. Reactions (Chlorobenzene): Aromatic nucleophilic substitution (replacement by $-OH$ group) and effect of nitro substituent. Benzyne Mechanism: KNH_2/NH_3 (or $NaNH_2/NH_3$). Reactivity and Relative strength of C-Halogen bond in alkyl, allyl, benzyl, vinyl and aryl halides.

Unit-III

Alcohols: Preparation: Preparation of $1^0, 2^0$ and 3^0 alcohols: using Grignard reagent, Ester hydrolysis, Reduction of aldehydes, ketones, carboxylic acid and esters. Reactions: Withsodium, HX (Lucas test), esterification, oxidation (with PCC, alk. $KMnO_4$, acidic dichromate, conc. HNO_3). Oppeneauer oxidation. Diols: (Upto 6 Carbons) oxidation of diols. Pinacol- Pinacolone rearrangement. Phenols: (Phenol case) Preparation: Cumene hydroperoxide method, from diazonium salts. Reactions: Electrophilic substitution: Nitration, halogenation and sulphonation. Reimer-Tiemann Reaction, Gattermann-Koch Reaction, Houben-Hoesch Condensation, Schotten-Baumann Reaction. Ethers (aliphatic and aromatic): Cleavage of ethers with HI.

Unit-IV

Carboxylic acid and its derivatives (aliphatic): Carboxylic acids (aliphatic and aromatic)

Preparation: Acidic and Alkaline hydrolysis of esters. Reactions: Hell–Vohlard - Zelinsky Reaction. Carboxylic acid derivatives (aliphatic): (Upto 5 carbons) Preparation: Acid chlorides, Anhydrides, Esters and Amides from acids and their inter conversion. Reactions: Comparative study of nucleophilicity of acyl derivatives. Reformatsky Reaction, Perkin condensation. Amines and Diazonium Salts: Amines (Aliphatic and Aromatic): (Upto 5 carbons), Preparation: from alkyl halides, Gabriel's Phthalimide synthesis, Hofmann Bromamide reaction. Reactions: Hofmann vs. Saytzeff elimination, Carbylamine test, Hinsberg test, with HNO_2, Schotten–Baumann Reaction. Diazonium salts: Preparation: from aromatic amines.Reactions: conversion to benzene, phenol, dyes.

Recommended books:

1. T. W. Graham Solomon: Organic Chemistry, John Wiley and Sons.
2. Peter Sykes: A Guide Book to Mechanism in Organic Chemistry, Orient Longman.
3. E. L. Eliel: Stereochemistry of Carbon Compounds, Tata McGrawHill.
4. I. L. Finar: Organic Chemistry (Vol. I & II), E. L. B.S.
5. R. T. Morrison & R. N. Boyd: Organic Chemistry, Prentice Hall.
6. Arun Bahl and B. S. Bahl: Advanced Organic Chemistry, S. Chand.

CHM-201L – Chemistry-II Lab

1. Detection of elements (N, S, Cl, Br, I) in organic compounds (containing upto two extraelements).

2. Separation of mixtures by Chromatography: Measure the Rf value in each case (combination of two compounds to be given).
   (a) Separation and identification of the components of a given mixture of 2 aminoacids (glycine,aspartic acid, glutamic acid, tyrosine or any other amino acid) by paper chromatography .
   (b) Separation and identification of the sugars present in the given mixture by paper chromatography. Purification of organic compounds by crystallization (from water and alcohol)and distillation. Criteria of Purity: Determination of melting and boiling points. Preparations: Mechanism of various reactions involved to be discussed.

3. Recrystallisation, determination of melting point and calculation of quantitative yields to bedone.
   (a) Bromination of Phenol/Aniline.
   (b) Benzoylation of amines/phenols.
   (c) Oxime and 2,4 dinitrophenylhydrazone ofaldehyde/ketone.

4. Any other practical/s found feasible by the teacherconcerned.

Recommended books:

1. A.I. Vogel: Textbook of Practical Organic Chemistry, 5th edition,Prentice-Hall.

2. F. G. Mann & B. C. Saunders, Practical Organic Chemistry, OrientLongman.

3. Advanced Practical Organic Chemistry, N. K. Vishnoi, Vikas Publishing House PvtLtd.

4. Laboratory manual in Organic Chemstry, R.K. Bansal, (WileyEastern).

GEC-201 – Environmental Science

Unit-I

Definition of Environment and Environmental Science, its scope and importance Concept of components of environment. Ecological Balance in Nature. Origin of the earth; Evolution of life. Biosphere: a brief account.

Unit-II

Ecosystem: Definition and types of ecosystems. Structure and Function. Food chains, Food webs and Ecological pyramids. Energy flow in an Ecosystem. Abiotic and biotic components of an aquatic and terrestrial ecosystem. Primary and secondary productivity: a brief account. Biotic interaction in ecosystem. Ecological Succession.

Unit-III

Biodiversity basic concept. India as mega biodiversity nation. Values of biodiversity. Threats to biodiversity. Hot spots of biodiversity. Conservation of biodiversity- in-situ and ex-situ. Concept of extinction threshold and extinction debt.

Unit-IV

Environmental Pollution. Definition, Causes, consequences and control of. Air pollution. Water pollution. Soil pollution. Noise pollution. Solid waste management. Wasteland reclamation. Sustainable development and causes of unsustainability. Environmental ethics. Global Warming, Climate Change, Acid rain and Ozone layer Depletion.

Recommended books:

1. Text Book of Environmental Studies, Bharucha, Erach, Universities Press (India), Hyderabad, 2005.
2. Environmental Sciences, Y. K. Singh, New Age International (P) Limited, Publishers, 2006.
3. Environmental Studies. Joseph, Benny, McGraw Hill Companies, 2005.

Integrated BSc – MSc Mathematics
Revised Syllabus

Semester-III

Course Structure				Marks			
S.No.	**Course Code**	**Course Title**	**Credits**	**CIA**	**MT**	**ET**	**TOTAL**
1	MTH-301	Plane and Solid Geometry	4	20	30	50	100
2	MTH-302	Real Analysis-I	4	20	30	50	100
3	MTH-303	Abstract Algebra-I	4	20	30	50	100
4	MTH-304	Statistics and Probability Theory	4	20	30	50	100
5	MTH-305	Matrix Algebra	4	20	30	50	100
6	CSC-301T	Programming in C	4	20	30	50	100
7	CSC-301L	Programming in C Lab	2	10	15	25	50
Total			26	130	195	325	650

Semester III

MTH-301 – Plane and Solid Geometry

Unit-I

Parabola: Equation of parabola, Equation of tangents and normals, Pole and polar, Equation of chord in terms of middle point, parametric equation of parabola.
Ellipse: Equation of ellipse, Equation of tangents and normals, Pole and polar, parametric equation of ellipse, Diameter and conjugate diameter.

Unit-II

Hyperbola: Equation of hyperbola, Equation of tangents and normals, Equation of hyperbola referred to asymptotes as axes, conjugate diameter, General second-degree equation in x and y, Tracing of conics.

Unit-III

Sphere: Equation of sphere, condition for two spheres to be orthogonal.
Cone: Equation of cone, condition for general second- degree equation to represent a cone, Necessary and sufficient conditions for a cone to have a three mutual perpendicular generators.

Unit-IV

Cylinder: Equations of cylinder, Enveloping cylinder of a sphere.
Central conicoids: tangens lines and tangent planes, Normal to conicoids at a point on it, Normal from a point to conicoids, shapes and features of the three central conicoids.

Recommended books:

1. Coordinate Geometry of conics, M. R. Puri, Kapoor publication.
2. Coordinate Geometry, Ram Ballabh,Malhotra publication.
3. Coordinate Geometry of two and three dimension, P. Balasubramanian, G. R Venkataraman.
4. Solid Geometry, M. R. Puri, Kapoor publication.
5. Analytical solid Geometry, shanty Narayan. S Chand and sons.

MTH-302 – Real Analysis-I

Unit-I

Sets and functions, countable and uncountable sets with examples. Real numbers; $\mathbb{Q}$ is coutable and $\mathbb{R}$ is uncountable. Least upper bound and greatest lower bound. Limit points, Bolzano-Weirastrass Theorem for bounded infinite sets. Open and closed sets. Dedekind's cuts, incompleteness of $\mathbb{Q}$ and completeness of $\mathbb{R}$. Archimedean property.

Unit-II

Sequences, convergence of a sequence, bounded and monotonic sequence. Nested Interval Theorem. Cauchy sequences, Cauchy's principle of convergence. Limit points of a sequence and Bolzano-Weirastrass Theorem for sequences. The limit superior and limit inferior of a sequence and related results. Sequential approach to continuity. The boundedness theorem and the intermediate value theorem for continuous functions. Uniform Continuity and related results.

Unit-III

Series; convergence and divergence of a series. Tests for convergence of positive term series: comparison test, p-test, ratio test, Rabes test, Gauss test, Logarithmic test, Cauchy's root test. Alternating series. Absolute convergence and conditional convergence. Leibnitz test.

Unit-IV

Inequalities: AM-GM inequality, Cauchy-Schwarz inequality, Chebyshev's inequality, Hölders and Minkowski inequality, convex and concave functions, Jensen's inequality, Bernoulli's inequality, Some applications involving inequalities.

Recommended books:

1. Mathematical Analysis, S.C. Malik and S. Arora, New Age International.
2. Calculus-I, T. Apostol, Wiley International.
3. Introduction to Real Anaysis, D. R. Sherbert and R. G. Bartle, Wiley International.
4. Inequalities: An Approach Through Problems, S. Venketachela, (TRIM) Hindustan Book Agency.
5. Methods of Real Analysis, R. Goldberg, Oxford and HBI Publishing.

MTH-303 – Abstract Algebra-I

Unit-I

Review of equivalence relations, equivalence classes and partitions. Permutations and their proprieties, cyclic, even and odd permutations. Integers: Well ordering principle, divisibility in integers, division algorithm, GCD and LCM. Fundamental theorem of arithmetic (statement only). Congruences, modular addition and multiplication with examples and properties. Euler$-\phi$ function and its properties.

Unit-II

Groups: basic properties, subgroups, examples and properties. Order of a group and order of an element in a group, examples and properties. Abelian and non-Abelian groups with examples and properties. Cyclic groups; examples, properties and related results. Subgroups generated by a subset of a group, examples and properties. Group of the roots of unity, modular groups. Klein's four group and its examples. Symmetric groups and their properties. Dihedral groups, matrix groups.

Unit-III

Cosets and related results, Lagrange's theorem and its applications. Normal subgroups; examples and properties, normalizers and centralizers. Quotient groups, examples, properties and related results. Group homomorphism, examples and properties. Isomorphism theorems on groups.

Unit-IV

Rings: Definition and examples, commutative and non-commutative rings with examples and properties, units and zero-divisors, nilpotent elements. Integral domains: examples and related results. Ring homomorphism: examples and properties. Ideals, examples and properties, prime and maximal ideals and related results. Quotient rings; properties and related results. Isomorphism theorem for rings, correspondence theorem. Fields and Division rings: examples and related results.

Recommended books:

1. Abstract Algebra, D. S. Dumit and R. M. Foote, Wiley Sons.
2. Topics in Algebra, I. N. Herstein, Wiley Sons.
3. Algebra, M. Artin, Pearson.

MTH-304 – Statistics and Probability Theory

Unit-I

Central tendency and measures of central tendency. Dispersion and measures of dispersion. Moments. Effect of change of origin and scale on moments. Pearson's β and γ coefficients. Skewness and measures of skewness. Kurtosis.

Unit-II

Correlation. Covariance and coefficient of correlation (Karl Pearson's coefficient of correlation). Limits for correlation coefficient. Effect of change of origin and scale on the correlation coefficient. Linear regression. Principle of least squares and lines of regression. Regression coefficients. Relationship between correlation and regression coefficients. Angle between the two lines of regression. Correlation coefficient of observed and estimated values.

Unit-III

Theory of Probability: origin, initial attempts at some mathematical rigor and their limitations. Kolmogorov's axiomatic development of probability theory: random (or statistical) experiments, sample space of a statistical experiment, event space of a random experiment, probability measure (or the probability set function), probability space. Monotone and subtractive properties of probability set function. Addition rule. Bonferroni's and Boole's inequality. Implication rule. Continuity of probability set function. Probability on finite sample spaces. Conditional probability and multiplication rule of probability. Independence of events. Prior and posterior probabilities. Bayes Theorem.

Unit-IV

Distribution functions. Set of discontinuities of a distribution function. Random Variables. Probability space induced by a random variable. Distribution function of a random variable. Discrete and continuous random variables. The probability mass and the probability density function. Functions of a random variable. Mathematical expectation of a random variable. Some special expectations. The moment generating function of a random variable. Some inequalities involving expectations: Markov's Inequality, Chebyshev-Bienayme inequality, Jensen's Inequality.

Recommended books:

1. S. C. Gupta and V. K. Kapoor, Fundamentals of Mathematical Statistics, Sultan Chand & Sons.

2. S. C. Gupta, Fundamentals of Statistics, Himalaya Publishing House.

3. Vijay K. Rohatgi and A.K. Md. Ehsanes Saleh, An Introduction to Probability and Statistics, John Wiley & Sons.

4. Robert V. Hogg, Joseph McKean and Allen T. Craig, Introduction to Mathematical Statistics, Pearson Education Inc.

MTH-305 – Matrix Algebra

Unit-I

Elementary matrices, Reduction of matrix to echelon form and normal form, row rank and column rank of matrices and their relations with rank, rank of the product of two matrices, row nullity and column nullity of matrices, relation between rank of a matrix and its adjoint. Homogeneous and non-Homogeneous system of linear equations. Determinants: Determinant by Laplace expansion and, alternating sums and permutations. Differentiation and integration of determinant.

Unit-II

Cayley-Hamilton theorem, Properties of Eigenvalues and Eigenvectors of matrices, Trace and determinant of a matrix and related results, Spectral mapping theorem, Eigenvalues of special matrices like Idempotent, Nilpotent and Involutary matrices, and their minimal equations. Algebraic and geometric multiplicity of Eigenvalues, Eigenvalues of similar matrices, Hermitian matrices, orthogonal matrices. Diagonalization of matrices and orthogonal diagonalization of symmetric matrices. Schur's theorem, Jordan canonical form.

Unit-III

Quadratic forms: matrix representation, KroneckerÂ"s reduction theorem, Lagrange's reduction theorem, reduction by orthogonal transformation of real quadratic forms. Positive definite, positive semi-definite, negative definite, negative semi-definite, determinant of positive definite. Rank, index and signature of a quadratic form. Necessary and sufficient conditions for definite form. If $A = [a_{ij}]$ is a positive definite matrix of order n, then $|A| \leq a_{11}a_{22}\cdots a_{nn}$. Cauchy-Bonet formula. Gram matrices. Hadamard's inequality, Gereshgorin disk theorem. If A and B are positive semidefinite matrices, then $|A+B| > |A|+|B|$. Trace inequalities.

Unit-IV

Special types of matrices: Idempotence, Nilpotence, Involution and projections. Necessary and sufficient conditions for a matrix to be idempotent, nilpotent and involutary. Orthogonal projection matrix and characterizations. Tridiagonal matrix: determinant and spectrum, Cauchy matrix, Circulant matrix, necessary and sufficient condition for a matrix to be circulant, Vandermonde matrices: determinant and inverse, Hadamard matrix and characterizations. Permutation matrix: characterizations and spectrum. Irreducibility of permutation matrices.

Recommended books:

1. Matrix Theory: Basic Results and Techniques, Second Edition, Fuzhen Zhang, Universitext Springer.
2. Matrices (Schaum Series), M. Spiegel, PHI.
3. Matrix analysis, Roger A. Horn and Charles R. Johnson, Cambridge.
4. The Theory of Matrices, F. R Gantmacher, University Press. Graphs and Application, John M Aldous, Springer.

CSC-301T – Programming in *C*

Unit-I

Programmer's Model of a Computer-Processor, Memory and working. Problem solving through C language: Introduction, History and Importance of C, Basic structure of C programs. C Tokens, Keywords and Identifiers, Data types, Variables and Constants. Declaration of variables and Value assigning. Defining Symbolic Constants. C Operators: Arithmetic, Relational, Logical, Assignment, Increment and Decrement, Conditional, Bitwise and special. Operator Precedence and associativity. C Expressions and their Evaluation. Type conversions.

Unit-II

Decision making and Control Structures in C: IF statement, IF ELSE statement, Nested IF ELSE statements, ELSE IF ladder, Switch statement, Conditional operator and GOTO statement. Looping and loop constructs in C: Basics and types, WHILE , DO, and FOR Constructs. Jumps in loops. Arrays: basics, 1-Dimensional, 2-Dimensional and n-Dimensional arrays. String -manipulation in C: Basics, declaration and initialization of string variables. Reading strings from users and writing strings to terminal. Arithmetic operations on character strings. Concatenation and comparison of strings. Built-in string manipulation functions in C. Basic Searching and Sorting techniques.

Unit-III

User-defined functions in C: Fundamentals and need, structure of C functions, return values and types, calling a function, different function categories in C, Function nesting in C, Recursion, scope and lifetime of variables in functions. Pointers in C: Fundamentals, declaring and initializing, accessing a variable through its pointer, pointer expressions, pointer arithmetic, pointers to arrays, strings and functions.

Unit-IV

Structures-Introduction and Basics. Programmatic Definition of a Structure, Assigning values to Structure members. Structure initialization, comparison of Structures. Arrays of structures, Arrays within structures, Structures within Structures, Structures and functions. Concept of Unions.

Recommended books:

1. Engineering Problem Solving with ANSI C, Delores M. Etter, PHI
2. Mastering C by Venugopal, Prasad, Tata McGraw Hill.
3. SPSS for Beginners, Vijay Gupta, VJ Books Incorporation.

CSC-301L – Programming in C Lab

Experiments in the lab are based on the contents of CSC-301T.

Integrated BSc – MSc Mathematics
Revised Syllabus

Semester-IV

Course Structure				Marks			
S.No.	**Course Code**	**Course Title**	**Credits**	**CIA**	**MT**	**ET**	**TOTAL**
1	MTH-401	Advanced Calculus	4	20	30	50	100
2	MTH-402	Linear Algebra	4	20	30	50	100
3	MTH-403	Real Analysis-II	4	20	30	50	100
4	MTH-404	Ordinary Differential Equations	4	20	30	50	100
5	MTH-405	Theory of Probability Distributions	4	20	30	50	100
6	CSC-401T	Data Structures	4	20	30	50	100
7	CSC-401L	Data Structures Lab	2	10	15	25	50
Total			26	130	195	325	650

Semester IV

MTH-401 – Advanced Calculus

Unit-I

Double integral over rectangles, volumes, double integrals and double Riemann sums with examples. The mid point rule, average value, iterated integrals. Double integral over general region with examples. Properties of double integrals. Double integral in polar coordinates with examples. Applications of double integrals.

Unit-II

Triple integrals, triple Riemann sums with examples, application of triple integrals, volume of hyper spheres, triple integral in cylindrical coordinates, evaluating triple integrals with cylindrical coordinates. Triple integrals in spherical coordinates, evaluating triple integrals with spherical coordinates. Change of variable in triple integrals.

Unit-III

Vector fields, expressing vector field in terms of its component functions. Gradient fields, line integrals with examples, line integral with respect to arc length, line integral of vector fields. Green's Theorem.

Unit-IV

Curl and divergence with examples, vector form of Green's theorem, Parametric surfaces and their areas, surfaces of revolution, tangent planes, surface area of a graph of a function, surface integral of a function over a surface, oriented surfaces, surface integrals of vector field. Stoke's Theorem, Divergence Theorem.

Recommended books:

1. Calculus, J. Stewart, Cengage learning.
2. Thomas' Calculus, J. Hass and M. D. Weir, Pearson.
3. Advanced Calculus, P. M. Fitzpatirick. AMS Publications.

MTH-402 – Linear Algebra

Unit-I

Vector spaces: definition, examples, linear combinations, linear dependence, linear independence, spanning sets. Subspaces. Linear spans, Row and column space of a matrix. Concept of a basis and related fundamental theorems: existence, replacement and extension theorems. Dimension of a finitely generated vector space.

Unit-II

Quotient spaces. Bases and dimension of a quotient space in a finite dimensional vector space. Sum and direct sum of subspaces. Systems of linear equations: Basic definitions, equivalent systems, elementary operations, systems in triangular and echelon form. Gaussian elimination. Number of linearly independent solutions of a consistent system of linear equations. Coordinates of a vector in a finite dimensional vector space.

Unit-III

Linear mappings (transformations), kernel and image of a linear mapping, rank and nullity of a linear mapping, singular and non-singular linear mappings, isomorphisms, operations with linear mappings. Algebra $A(V)$ of linear mappings and related results.

Unit-IV

Matrix representation of linear transformations acting between finite dimensional vector spaces. Change of basis matrices and related results. Similarity of matrices of a linear operator on a finite dimensional vector space. Characteristic and minimal polynomials, Cayley-Hamilton Theorem. Eigen values, eigen vectors and diagonalization.

Recommended books:

1. Linear Algebra, K. Hoffman and R. Kunze, Pearson Education.
2. Elementary Linear Algebra, Howard Anton, Wiley.
3. Linear Algebra, Serge Lang, Springer.
4. Linear Algebra, Seymour Lipchutz and Mark Lipson, Tata McGraw Hill.

MTH-403 – Real Analysis-II

Unit-I

Riemann Integration: partition of an interval, refinement of a partition, Darboux sums. upper and lower Riemann integrals. Necessary and sufficient condition for existence of a Riemann integral. Algebra of Riemann integrable functions. Integrability of $|f|$. Fundamental theorem and the mean value theorem of integral calculus .

Unit-II

Riemann-Stieltjes integral, definition and existence of the integral, partitions and their refinement, conditions of integrability, algebra of integrable functions, reduction of Riemann-Stieltjes to Riemann integral, integration and differentiation: Fundamental Theorem of Calculus. Integral as a limit of sums and mean value theorems. Integration by parts.

Unit-III

Improper integrals: integration of unbounded functions with finite range of integration, comparison tests for convergence, Cauchy's test, absolute convergence, integration of bounded functions with infinite range, convergence of integrals of unbounded functions with infinite limits, integration of product of functions, Abel's and Drichlet's tests.

Unit-IV

Uniform convergence of sequence and series of functions: Point-wise convergence, uniform convergence on an interval, Cauchy's criterion for uniform convergence M_n-test for uniform convergence for sequence, Weirestrass's M-test, Abel's and Dirchlet's tests for uniform convergence of series. Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, Weirestrass approximation theorem. Conditional and absolute convergence of series, Dirichlet's and Reimann's Rearrangement theorems for series.

Recommended books:

1. Robert G. Bartle and Donald R. Sherbert, Introduction to Real Analysis, 4th Edition, Wiley India Pvt. Ltd.

2. Principles of Mathematical Analysis, S.C. Malik and S. Arora, New Age Internationals.

3. Mathematical Analysis, T. M. Apostal, Narosa.

4. Principles of Mathematical Analysis, W. Rudin, Tata McGraw Hill.

MTH-404 – Ordinary Differential Equations

Unit-I

Differential equations of first order but not of first degree, equations solvable for x, y, z, p. Clairaut's form, equations reducible to Clairaut's form. Singular solutions, extraneous loci: tac locus, node locus and cusp locus, p-discriminant and c-discriminant. Method of variation of parameters for linear equations of second order.

Unit-II

Initial value problems of first order ODE, Ascoli−lemma, Fundamental theorem for the existence and uniqueness of solution of an IVP, Picard's theorem on the existence and uniqueness of solutions to an initial value problem. Method of successive approximations. Sturm-Lioville Problem (SLP) for second order differential equation, properties of eigenvalues and eigenfunctions of SLP.

Unit-III

Linear dependence and independence of solutions of an ODE, properties of Wronskian of the solutions of an ODE and Abel's formula. Method of construction of Green's function of one dimension. Simultaneous equation $\frac{dx}{P} = \frac{dx}{Q} = \frac{dz}{R}$ and its solutions by use of multipliers and a second integral found by the help of first. Total differential equations $Pdx + Qdy + Rdz = 0$. Necessary and sufficient condition for integrability of an equation. Geometric interpretation of the $Pdx + Qdy + Rdz = 0$. Simultaneous system of ODE in matrix form.

Unit-IV

Solution in Series: (i) Roots of an indicial equation, un-equal and differing by a quantity not an integer. (ii) Roots of an indicial equation, which are equal. (iii) Roots of an indicial equation differing by an integer making a coefficient infinite. (iv) Roots of an indicial equation differing by an integer making a coefficient indeterminate. Legendre's equation and Bessel's equation, and their recurrence formulae.

Recommended books:

1. Shepley L. Ross, Differential Equations, John Wiley and Sons.
2. Simmons , Differential Equations with Applications and Historical Notes.
3. M. D. Rainsinghania, Ordinary and Partial Differential Equations.
4. H.T.H. Piaggio, Differential Equations, CBS Publishers and Distributors, New Delhi.

MTH-405 – Theory of Probability Distributions

Unit-I

Standard univariate discrete distributions: Uniform distribution on n points, Bernoulli distribution, Binomial distribution, Negative binomial distribution (or waiting-time distribution), Hypergeometric distribution, Negative hypergeometric distribution, Poisson distribution.

Unit-II

Some special univariate continuous distributions: Uniform (or rectangular) distribution, gamma distribution, two parameter gamma distribution, chi-square distribution, beta distribution, Cauchy distribution, the normal distribution.

Unit-III

Multiple (or n-dimensional) random variables, with special reference to 2-dimensional random variables. Distribution function of a 2-dimensional (or bivariate) random variable. Bivariate random variables of discrete and continuous type. Joint probability mass and density functions. Marginal probability mass and density functions. Marginal distribution functions. Conditional probability mass and density functions. Conditional distribution functions. Independent Random variables. Identically distributed random variables.

Unit-IV

Expectations and moment generating functions of multiple random variables. Some multivariate distributions: Multinomial distribution, Bivariate normal distribution, Student's t-distribution, F-distribution.

Recommended books:

1. Vijay K. Rohatgi and A.K.Md.Ehsanes Saleh, An Introduction to Probability and Statistics, John Wiley & Sons.
2. Robert V. Hogg, Joseph McKean and Allen T. Craig, Introduction to Mathematical Statistics, Pearson Education Inc.
3. S. C. Gupta and V. K. Kapoor, Fundamentals of Mathematical Statistics, Sultan Chand & Sons.

CSC-401T – Data Structures

Unit-I

Data structures-Introduction, Classification. Concept of ADT's. Linked Lists-basics, Linked Lists vs Arrays, types of Linked Lists. Singly Linked List- Implementation-static and pointer, traversing a List, searching for a value, insertion of a new node, deletion of a node. Circular Linked List-inserting a new node, deleting a node. Doubly Linked Lists-inserting a new node, deleting a node. Applications of Linked Lists-Polynomial representation.

Unit-II

Stacks-Introduction. Representation of Stacks-Array and Linked List. Operations on Stacks-Push, POP and Peek. Applications of Stacks-Evaluations of arithmetic expressions. Queues-Introduction, representation-Array and Linked List. Types of Queues-Circular, Deques and Priority Queues. Applications of Queues.

Unit-III

Trees-Introduction and basic Terminology. Types of Trees-Overview. Binary Trees- Creation of Binary Trees and Traversing a Binary Tree-Pre-Order traversal, In-order Traversal and Post-Order Traversal. Binary Search trees-Introduction, Operations- Insertion, Deletion and searching for a Key. Applications of Trees-Overview.

Unit-IV

Graphs-Introduction and Basic Graph terminology. Representation of Graphs- Adjacency Matrix representation and Adjacency List representation. Graph traversal Algorithms-Breadth-First Search and Depth-First Search. Shortest Path Algorithms- Minimum Spanning Trees, Dijkistra's Algorithm and Warshall's Algorithm. Applications of Graphs-Overview.

Recommended books:

1. Fundamentals of Data Structures, Sahni Horowitz, Universities Press.
2. Data Structures Using C and $C++$, Y.L. Moshe, J. Augenstein.

CSC-401L – Data Structures Lab

Experiments in the lab are based on the contents of CSC-401T.

Integrated BSc – MSc Mathematics
Revised Syllabus

Semester-V

Course Structure				Marks			
S.No.	Course Code	Course Title	Credits	CIA	MT	ET	TOTAL
1	MTH-501	Complex Analysis-I	4	20	30	50	100
2	MTH-502	Abstract Algebra-II	4	20	30	50	100
3	MTH-503	Topology-I	4	20	30	50	100
4	MTH-504	Operation Research	4	20	30	50	100
5	MTH-505	Number Theory	4	20	30	50	100
6	MTH-506	Numerical Analysis	4	20	30	50	100
Total			24	120	180	300	600

Semester V

MTH-501 – Complex Analysis-I

Unit-I

Functions of a complex variable, Limits, Continuity, Differentiability, Cauchy-Riemann Equations and their applications, Analytic function, Harmonic function, The functions like $e^z, \sin z, \cos z$ and the complex logarithm. Contour integral, Cauchy's theorem, Cauchy-Goursat's theorem, Cauchy's integral formula, Higher order derivatives, Morera's theorem, Cauchy's inequality, Liouville's theorem and its applications, Winding numbers-index of a point with respect to a closed curve.

Unit-II

Power Series, Radius of convergence of a power series, Cauchy's-Hadamard formula for finding radius of convergence, Taylor theorem, Taylor's series, Expansion of analytic functions in a power series, Laurent's series, isolated and essential singularities. Behavior of functions at infinity, Casorati-Weirestrass's Theorem.

Unit-III

Bilinear transformations- Properties and Classification, Fixed Points, Cross Ratios, Inverse Points and Critical Points. Conformal mapping, Mappings of: Upper half plane on to unit disc, unit disc onto unit disc, left half plan on to unit disc, Circle onto circle. The transformations: $w = \sqrt{z}, z^2, \frac{1}{z}, (z + \frac{1}{z})$.

Unit-IV

Residues: Cauchy's residue theorem and its applications, Calculation of residues, Evaluation of definite integrals by the method of residues, Parseval's identity. Infinite products, Convergence and divergence of infinite products.

Recommended books:

1. L. Ahlfors, Complex analysis, Tata McGraw Hill.
2. Richard Silverman, Introductory Complex Analysis, Dover Publications.
3. S. Ponnusamy, Foundations of Complex Analysis, Narosa.
4. Z. Nihari, Conformal Mappings, Dover Publications.
5. E.C. Titchmarsh, Theory of Functions, Oxford Science Publications.

MTH-502 – Abstract Algebra-II

Unit-I

Group action, stabilizers and orbits, Cayley's theorem, class equation and its applications, Conjugacy in S_n and cycle decomposition. Simple groups, simplicity of alternating groups. Automorphisms: examples and related results, inner automorphism and related results.

Unit-II

Direct sums and direct products, p-primary groups, Primary decomposition theorem, pure subgroups, Basis Theorem, Fundamental theorem of finite abelian groups and its applications.

Unit-III

Cauchy's theorem of finite groups, Sylow theorems and their applications: groups of order pq, p and q are primes and $p < q$, groups of order $30, 12, 60$ and groups of order p^2q, where p and q are distinct primes. Projective unimodular groups.

Unit-IV

Maximal subgroups and p-groups, nilpotent groups exampls and related results, commutators and lower central series, solvable groups and derived series, composition series for groups, Zessenhaus's Lemma, Schreier Refinement Theorem and Jordan Holder Theorem.

Recommended books:

1. Advanced Modern Algebra, Joseph J. Rotman, Graduate studies in Mathematics, AMS.
2. Abstract Algebra, D. S. Dumit and R. M. Foote, Wiley.
3. Topics in Algebra, I. N. Herstein, Wiley.
4. Algebra, M. Artin, Pearson.

MTH-503 – Topology-I

Unit-I

Review of countable sets, uncountable sets and Schroeder-Bernstein theorem, Metric spaces: definition and examples, Interior point and limit point, Open sets, closed sets, Convergence, completeness in metric spaces, Continuous functions, space of continuous functions and completion of a metric space.Cantor's intersection theorem.

Unit-II

Nowhere dense set, first category and second category, F_σ set, G_δ set, oscillation of $f : R \to R$ at a point and its relationship with continuity, set of discontinuities of $f : R \to R$. Baire's category theorem and its application to the (i) non-existence of a real valued function on R which is precisely continuous at rationals. (ii) impossibility of approximating the characteristic function of rationals on $[0, 1]$ by a sequence of continuous functions. Uniformly continuous mappings with examples and counter examples. Extending uniformly continuous mappings. Banach's contraction principle with applications to the inverse function Theorem in $\mathbb{R}$.

Unit-III

Topological spaces: Definition and examples, open sets, closed sets, discrete and indiscrete topologies, finite and co-finite topologies, coarser and finer topologies, interior and closure, Kuratowski's axioms, basis and sub-basis for a topology, lower limit topology, the order topology, subspace topology. Continuous mappings and their characterizations, homeomorphisms, the pasting lemma. Nets in topological spaces, convergence of nets and continuity in terms of nets.

Unit-IV

Concept of first countability and second countability, separability and their relationship. Weak topology generated by family of function with examples, product topology. Compactness in topological spaces, finite intersection property, compactness in metric spaces and Balzano Weirstrass property, Heine-Borel Theorem in R. Connected spaces: separation, connected subspaces of the real line, intermediate value theorem, path connectedness, components and local connectedness.

Recommended books:

1. Introduction to Topology and Modern Analysis, G. F. Simmons, Tata McGraw Hill Education.
2. Topology, J. R. Munkres, Pearson.
3. General Topology, J. L. Kelley, Springer.

MMT-504 – Operations Research

Unit-I

Introduction to Operation Research, Linear Programming Problem: mathematical formulation for LP problems and Graphical method. Definitions: Slack and Surplus variables, matrix form of LPP, feasible solution, basic solution, basic feasible solution, degenerate and non-degenerate solution. Redundant constraint, reduction of a feasible solution to a basic feasible solution. Convex sets: the set of all feasible solutions to an LPP is a convex set, Extreme point theorem, Simplex method, Flowchart for simplex method.

Unit-II

Two Phase method, Big-M method for solving LP problems with optimal solution, infeasible solution, degenerate solution, unbounded solution and multiple optima solution, Concept of duality in Linear programming, formation of Primal-Dual problems, dual of a dual is primal. Weak duality theorem, Strong duality theorem, examples.

Unit-III

Transportation Problems: Mathematical formulation, necessary and sufficient condition for the existence of feasible solution, loops in TP problems. Methods for initial basic feasible solution: North-West corner method, Least cost method, Vogel's approximation method, U.V method. Assignment problems: Hungarian method, Sensitivity Analysis: changes in the coefficients of the objective function, adding and deletion of a constraint and a variable.

Unit-IV

Project Management: construction of network, Construction by critical path method (CPM) and by PERT (probability consideration in PERT). Game theory: Two person zero sum games, games with pure strategies, mixed strategies, Min. Max. principle, dominance rule, solution of $2 \times 2, 2 \times n, m \times 2$ games.

Recommended books:

1. Operations Research: An Introduction, Hamdy Taha, Pearson.
2. Operations Research, Kanti Swarup, P. K. Gupta and Man Mohan, Sultan Chand and Sons.
3. Operations Research, Richard Bronson and Govindasami Naadimuthu, Schaum's Outline Series, Tata McGraw Hill.

MTH-505 – Number Theory

Unit-I

Introduction to numbers, GCD is the linear combination of two integers, other related properties, relatively prime integers, Euclidean algorithm. Sequence of primes, Fundamental theorem of Arithmetic, LCM, EuclidÂ's second theorem, infinitude of primes, arbitrary large gaps in the series of primes, no polynomial $f(x)$ with integers can be a prime for all x, Goldbach conjecture, DirichletÂ's theorem. Fermat numbers. Radix representation. Linear Diophantine equations, positive solutions.

Unit-II

Congruences and properties, Arithmetic functions, multiplicative functions, basic examples. Euler's ϕ (Toient) function, its properties. Complete Residue System, Reduced Residue System and related results. Fermat's theorem, Euler's theorem, Wilson's theorem and their applications and related results.

Unit-III

System of linear Congruences, Chinese Remainder theorem, solution of polynomial Congruences, a necessary and sufficient condition that the congruence $ax \equiv b(mod\ m)$ is solvable and related results, Congruences of higher order, Congruences with prime power, prime moduli and related results, Lagrange's theorem.

Unit-IV

Division Algorithm for polynomials, Factor theorem, its Generalization, equivalence of polynomials, form of degree n, Chevalley's theorem, Warning's theorem, Quadratic forms over a field of characteristics $\neq 2$, equivalent quadratic forms, Direct sum of quadratic forms, Witt's theorem, Representation of field elements, an integer is the sum of two, three and four squares.

Recommended books:

1. An introduction to the Theory of Numbers, I. Niven, H.S. Zuckerman and H. L. Montgomery, Wiley; fifth Edition.
2. Topics in Number Theory, W. J. Leveque, Dover Publications; vols 1 and 2 edition.
3. Elementary number Theory, David M. Burton, McGraw Hill Education, 7th Edition.
4. Elementary Number Theory with Applications, Thomas Koshy, Academic Press; 2nd Edition.

MTH-506 – Numerical Analysis

Unit-I

Error estimation, solution of algebraic, transcendental and polynomial equations. Solution by Bisection method, iteration method, Method of falsi position, Newton-Raphson method, rate of convergence of Newton-Raphson method and Secant method. Solution of the system of nonlinear equations.

Unit-II

Interpolation and Langrange polynomial, Finite differences, Forward differences, Backward differences, Central differences, Symbolic relations and separation of symbols, detection of errors. Newton's formulae for interpolation, Lagrange's interpolation formula, its errors, Hermite's interpolation formula, Gauss forward and backward formulae.

Unit-III

Numerical differentiation and its errors, Cubic spline method. Numerical integration, trapezoidal rule, Simpson's 1/3 and 3/8 rules. Evaluation of double integrals by trapezoidal and Simpson's rule. Eigenvalue problem, eigenvalues of symmetric Tridiagonal matrix. Legendre polynomials, linearly independent functions, orthogonal functions

Unit-IV

Numerical solution of ordinary differential equations, solution by Taylor's series, Picard's method, Euler's method, modified Euler's method, Runge-Kutta methods. Numerical solution of integral equations, method of degenerate Kernel's.

Recommended books:

1. Numerical methods, M. K. Jain, S. R. K. Iyengar and R K Jain, New AGE; 6th edition.
2. Introductory Methods of Numerical Analysis, S.S. Sastary, Prentice Hall India Learning Private Limited; fifth edition.
3. Applied Numerical methods, C.F. Gerald and P.O. Wheatley, Pearson Addison Wesley, Greg Tobin; 7th edition.
4. Numerical solution of differential equations, M. K. Jain, New AGE International Publishers; fourth edition.

Integrated BSc – MSc Mathematics
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Semester-VI

Course Structure				Marks			
S.No.	**Course Code**	**Course Title**	**Credits**	**CIA**	**MT**	**ET**	**TOTAL**
1	MTH-601	Calculus of Transform	4	20	30	50	100
2	MTH-602	Abstract Algebra-III	4	20	30	50	100
3	MTH-603	Multivariate Calculus	4	20	30	50	100
4	MTH-604	Graph Theory	4	20	30	50	100
5	MTH-605	Lattices and Ordered Structures	4	20	30	50	100
6	CSC-601T	Computational Mathematics	4	20	30	50	100
7	CSC-601L	Computational Mathematics Lab	2	10	15	25	50
Total			26	130	195	325	650

Semester VI

MTH-601 – Calculus of Transform

Unit-I

Periodic Functions. Piecewise Continuous Functions. Fourier Series and the need for Fourier Series. Dirichlet Conditions. Odd and Even Functions. Half-Range Fourier Sine and Cosine Series. Parseval's Identity. Uniform Convergence. Integration and Differentiation of Fourier Series.

Unit-II

Fourier transform, inverse Fourier transform, Fourier sine and cosine transforms and their inversion, properties of Fourier transforms, Fourier transform of the derivative, convolution theorem, discrete Fourier transform and fast Fourier transform and their properties, applications of Fourier transform in partial differential equations with special reference to heat and wave equation.

Unit-III

Laplace transform-definition, functions of exponential order, sufficient conditions for existence of Laplace transform, linearity property, first and second translation (shifting property), Laplace transform of derivatives, Laplace transform of integrals. Laplace transform of some elementary functions, methods of finding Laplace transform. Laplace transform of special functions

Unit-IV

Definition and uniqueness of inverse Laplace transform, Lerch's theorem, some properties of inverse Laplace transform, inverse Laplace transform of derivatives and integrals, the convolution property, methods of finding inverse Laplace transform, the complex inversion formula, the Heaviside expansion formula. Applications to differential equations: ordinary differential equations with constant coefficients and with variable coefficients.

Recommended books:

1. An Introduction to Laplace Transforms and Fourier Series, Phil Dyke, Springer Undergraduate Mathematics Series.
2. The Laplace Transform Theory And Applications, Joel L. Schiff, Springer Undergraduate Mathematics Series.
3. Fourier Analysis Theory And Problems, Murray R. Spiegel, Schaum's Outline Series.
4. Laplace Transforms, Murray R. Spiegel, Schaum's Outline Series.

MTH-602 – Abstract Algebra-III

Unit-I

Review of rings: polynomial rings, formal power series rings, matrix rings, group rings, quadratic integer rings and their properties; prime ideals, maximal ideals, sum, product, colon and radicals of ideals, principal ideals, comaximal ideals and Chinese-Remainder Theorem.

Unit-II

Field of fractions and embedding theorem. Euclidean domains; examples and related results, existence of GCD and related results in Euclidean domains, universal side divisors and their properties. Principal ideal domains (PIDs) examples and related results, Dedekind-Hasse norm, PIDs which are not Euclidean domains.

Unit-III

Irreducible elements, primes, examples and related results, Unique Factorization Domains (UFDs), examples and related results; PID implies UFD. Factorization in Gaussian integers: Fermat's theorem on sum of squares and its applications .

Unit-IV

Polynomial rings in severable variables, polynomial rings over fields. Unique factorization in polynomial rings: Gauss's Lemma and Gauss's Theorem. Irreducibility criteria for polynomials, Eisenstein's Criterion. Hilbert's Basis Theorem.

Recommended books:

1. Advanced Modern Algebra, Joseph J. Rotman, American Mathematical Society.
2. Abstract Algebra, D. S. Dumit and R. M. Foote, Wiley Sons.
3. Topics in Algebra, I. N. Herstein, Wiley Sons.
4. Algebra, M. Artin, Pearson.

MTH-603 – Multivariate Calculus

Unit-I

Functions of two variables, limit of a function, algebra of limits, repeated integrals, continuity, higher order partial derivatives, differentiability, sufficient condition for differentiability, change in the order of the partial derivatives, Young's Theorem, Schwarz's Theorem, chain rule, differentiation under integral sign.

Unit-II

Functions of several variables, limit, continuity, partial derivatives, directional derivatives, total derivative, continuity and their relation. Necessary and sufficient condition for differentiability. Matrix of linear transformation of differentiable functions at a point, chain rule, mean value theorems for differentiable functions.

Unit-III

Partial derivatives of higher order, sufficient condition for equality of mixed partials, Taylor's theorem for functions from $\mathbb{R}^n$ to $\mathbb{R}$. Implicit and inverse function theorem.

Unit-IV

Level curves, gradient vector and its significance, tangent plane and linear approximation. Extreme points and extreme values for function from $\mathbb{R}^2$ to $\mathbb{R}$: Necessary and sufficient conditions for extreme points. Lagrange multipliers, Isoperimetric problems.

Recommended books:

1. Mathematical Analysis, S. C. Malik, S. Arora, New Age International Publishers.
2. Mathematical Analysis, T. M. Apostol, Narosa.
3. Calculus, J. Stewart, Cengage Learning.

MTH-604 – Graph Theory

Unit-I: Introduction to Graphs

Types of graphs, graph isomorphism, walks, paths and cycles, Konig's theorem on bipartite graphs, graph operations, Konigsberg bridge problem, Euler graphs and Euler's theorem, Hamiltonian graphs, Dirac's theorem, Ore's theorem, Hamiltonian cycles in K_n (n odd), connectedness, signed graphs, balanced signed graphs.

Unit-II: Degree sequences and Trees

Degree sequences, Wang and Klietman theorem, Havil-Hakimi theorem, Hakimi's theorem, Erdos-Gallai theorem, Degree sets, trees and their properties, binary and spanning trees, complexity of a graph, Labelled trees, Cayley's Theorem, fundamental cycles.

Unit-III: Connectivity and Planarity

Cut sets, vertex and edge connectivity, Whitney's theorem, properties of a bond, block graphs, planar graphs, Euler's formulla, Kuratowski's theorem, geometric dual, Whitney's theorem, regular polyhedra.

Unit-IV: Graph Matrices

Incidence matrix $A(G)$, modified incidence matrix A_f, cycle matrix $B(G)$, fundamental cycle matrix B_f, cut-set matrix $C(G)$, fundamental cut set matrix C_f, relation between A_f, B_f and C_f , path matrix, adjacency matrix, matrix tree theorem, Laplacian Matrix.

Recommended books:

1. A Text Book of Graph Theory, R. Balakrishnan, Ranganathan, Springer- Verlag.
2. An Introduction to Graph Theory, S. Pirzada, Universities Press, Orient Blackswan, Hyderabad, India, (2012).
3. Extremal Graph Theory, B. Bollobas, Academic Press.
4. Graph Theory with Applications to Engineering and Computer Science, Narsingh Deo, Prentice Hall.
5. Introduction to Graph Theory, Gary Chartrand and Ping Zhang, Tata McGraw Hill.

MTH-605 – Lattices and Ordered Structures

Unit-I

Partial orders and equivalence relations. Dual order, total order, lexicographic order. Chains and anti-chains, Zorn's lemma. Well-Ordering Principle and its equivalent forms. Principle of Duality, top and bottom elements, Hasse diagram. Order-preserving mappings principal down and up sets. Residuated mappings and related results.

Unit-II

Closures: dual closure, fixed points, bicomplete ordered sets. Isomorphisms of ordered sets: order isomorphism, dually isomorphic ordered sets. Galois connections: left and right annihilators. Semigroups of residuated mappings: Generalised Baer semigroup and related results.

Unit-III

Semilattices and lattices, lower and upper bounds of lattices. Down-set lattices, maximal and minimal elements morphisms. Sublattices: ideals, filters and related results. Lattice morphisms and related results.

Unit-IV

Complete lattices: definition, examples and related results. Knaster theorem and Bernstein theorem. Embedding of an ordered set: Dedekind–MacNeille Theorem. Baer semigroups: Definition and related results. Complemented lattices: examples and related results. Boolean algebras and Boolean rings.

Recommended books:

1. Lattices and Ordered Algebraic Structures, T.S. Blyth, Springer.
2. Lattice Theory, G. Birkhoff, American Mathematical Society.
3. Sets, lattices and Boolean Algebras, J.C. Abbott, Allyn & Bacon, Boston.

CSC-601T – Computational Mathematics

Unit-I

Computing in Mathematics: Need, domain and overview of the discipline of Computational Mathematics. Problem solving and the notion of "Algorithm": Origin, definition and properties. Relation between an algorithm and a computer program. Understanding programming, conversing in Python for Mathematical computing: Variables, data types and operators. Decision control structures, looping and supporting constructs. Working with text: Strings, pattern matching and regular expressions. Functions and recursion.

Unit-II

Introduction to Numpy. Matrices & Linear algebra with Numpy: Addition, Multiplication, Transpose, accessing individual elements, rows & columns of a Matrix. Performing Slicing on Matrices. Matrix and Vector products: dot product of vectors and Matrices, inner product of two matrices, outer product of two vectors, matrix product, determinant, trace and inverse of a matrix. Performing Tensor product along specified Axes, Raising a square matrix to nth power. Decompositions: Cholskey decomposition, QR factorization and single value decomposition of a matrix. Eigen Value problem: Eigen values and right Eigen vectors of a square matrix, Eigen values and Eigen vectors of a complex Hermitian or a symmetric matrix. Solutions of systems of linear equations: Gauss Elimination Method.

Unit-III

Introduction to SciPy. Numerical solutions of algebraic non-linear equations: Iterative methods- Bisection method, Newton-Raphson method, Secant method and Fixed-Point iteration method. Matplotlib: Introduction, basic graphs and visualizations. SAGE: Introduction: Basic Group Theory with SAGE.

Unit-IV

Mathematica: Introduction, detailed exploration of notions of calculus of one variable and simple multivariable calculus. Linear algebra with Mathematica. Cryptography: Overview and basics of Cryptology, basic notions of Symmetric and Public-Key cryptography. Using built-in functions in Mathematica to encrypt an expression with symmetric and asymmetric encryption, generating and using symmetric and asymmetric keys in Mathematica.

Recommended books:

1. Fundamentals of Computers, V. Rajaraman, PHI.
2. Introduction to Computers, Peter Norton, Tata McGraw Hill.
3. Boolean Algebra and Switching Circuits, Eliot
4. Mendelson: Schaum's Outline Series. Microsoft Office 2019, Microsoft Press.

CSC-601L – Computational Mathematics Lab

Experiments in the lab that are based on the contents of MTH-606.

Integrated BSc – MSc Mathematics
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Semester-VII

Course Structure				Marks			
S.No.	Course Code	Course Title	Credits	CIA	MT	ET	TOTAL
1	MTH-701	Differential Geometry	4	20	30	50	100
2	MTH-702	Abstract Algebra-IV	4	20	30	50	100
3	MTH-703	Functional Analysis-I	4	20	30	50	100
4	MTH-704	Measure Theory-I	4	20	30	50	100
5	MTH-705	Advanced Number Theory	4	20	30	50	100
6	SS	Soft Skills (Open Elective)	4	20	30	50	100
Total			24	120	180	300	600

Semester VII

MTH-701 – Differential Geometry

Unit-I

Curves, Planar curves, Regular curves, Parameterization of curves, Arc-length, and arc-length is independent of parameterization, unit speed curves. Plane curves: Curvature of plane curves, osculating circle, centre of curvature. Computation of curvature of plane curves. Directed curvature, Fundamental theorem for plane curves.Examples: Straight line, circle, ellipse, tractrix, evolutes and involutes. Space curves: tangent vector, unit normalvector and unit binormal vector to a space curve. Curvature and torsion of a space curve. The Frenet–Serret Theorem. First Fundamental theorem of space curves. Intrinsic equation of a curve. Computation of curvature andtorsion.

Unit-II

A brief review of continuity and differentiability and revisiting Inverse function theorem (statement only), Surfaces, Regular surfaces, coordinate charts. Change of parameters; Differentiable functions on surfaces, The tangent plane, Differential of a map, First Fundamental Form, Orientation of surfaces, line element and its invariance under change of coordinates, angle between two curves. Geometrical definition of area.

Unit-III

Curvature of a Surface: Normal curvature, Euler's work on principal curvature, Qualitative behaviour of a surface near a point with prescribed principal curvatures. The Gauss map and its differential, together with its properties. Second fundamental form. Gaussian curvature, Mean curvature Weingarten equation. Ruled surfaces and minimal surfaces, Isothermal coordinates. Surfaces with constant positive or negative Gaussian curvature. Gaussian curvature in terms of area. Line of curvature, Rodrigues's formula for line of curvature, Equivalence of Surfaces: Isometry between surfaces, local isometry, and characterization of local isometry.

Unit-IV

Christoffel symbols. Expressing Christoffel symbols in terms of metric coefficients and their derivative. Theorema Egregium (Gaussian curvature is intrinsic). Isometric surfaces have same Gaussian curvatures at corre- sponding points. Gauss equations and Manardi Codazzi equations for surfaces. Fundamental Theorem for regular surface. (Statement only). Geodesics: Geodesic curvature, Geodesic curvature is intrinsic, Equations of Geodesic, Geodesic on surfaces of revolution and sphere. Geodesic as distance minimizing curves. Gauss-Bonnet theorem (statement) and its implication for geodesic triangle on sphere.

Recommended books:

1. Elementary Differential Geometry, Andrew Pressly, Springer.
2. Elementary Differential Geometry, Barret O'Neil, Academic Press.
3. Geometry from a Differentiable Viewpoint, John Mc Cleary, Cambridge Univ. Press.

MTH-702 – Abstract Algebra-IV

Unit-I

Free abelian groups, free groups, presentations and related results, Von Dyck's Theorem, existence of group of generalized Quaternions ($\mathbb{Q}_n$), rank of a free group, Schreier transversal, Neilson-Scheier Theorem on free groups.

Unit-II

Modules: basic definitions and examples, submodules, quotient modules and module homomorphisms, isomorphisms theorem on modules, generation of modules and direct sums, simple modules, examples, properties and related results, Schur's lemma and its applications, annihilators, torsion elements, torsion and torsion free modules.

Unit-III

Exact, short exact and split exact sequences. Four and Five lemma. Basics of categories and functors. Free Modules: Properties and related results, rank of a module and related results. Tensor product of Modules with basic properties and applications, projective, injective and flat modules, Modules with chain conditions: Noetherian and Artinian modules.

Unit-IV

Fundamental theorem for finitely generated modules (existence and uniqueness), primary decomposition theorem, elementary divisors and invariant factors, rational canonical form, converting $n \times n$ matrix into rational canonical form, Jordan canonical form, converting $n \times n$ matrix into Jordan canonical form, changing one canonical form into another.

Recommended books:

1. Abstract Algebra, D. S. Dumit and R. M. Foote, Wiley Sons.
2. University Algebra, N. S. Gopalakrishnan, New Age International Publishers.
3. Advanced Modern Algebra, Joseph J. Rotman, Graduate Studies in Mathematics.

MTH-703 – Functional Analysis-I

Unit-I

Banach Spaces: Definition and examples, subspaces, quotient spaces, continuous linear operators and their characterization, completeness of the space $L(X, Y)$ of bounded linear operators (and its converse), incompleteness of $C[a, b]$, under the integral norm, finite dimensional Banach spaces, Equivalence of norms on finite dimensional space and its consequences, dual of a normed linear space, Hahn Banach theorem (extension form) and its applications, complemented subspaces, duals of C_0, l_p $(p \geq 1), C[a, b]$.

Unit-II

Uniform boundedness principle and weak boundedness, dimension of an n-dimensional Banach space, conjugate of a continuous linear operator and its properties, Banach-Steinhauss Theorem, Open Mapping and Closed Graph Theorems, counterexamples to Banach-Steinhauss, Open Mapping Theorem and Closed Graph Theorems for incomplete domain and range spaces, separable Banach spaces and the separability of some concrete Banach spaces $(C_0, C[0, 1], l_p, p \geq 1)$, reflexive Banach Spaces, closed subspace and the dual of a reflexive Banach space, examples of reflexive and non-reflexive Banach spaces.

Unit-III

Hilbert spaces: Definition and examples, Cauchy's Schwartz inequality, Parallelogram law, orthonormal systems, Bessel's inequality and Parseval's Identity for complete orthonormal systems, Riesz-Fischer Theorem, Gram Schmidt process, orthonormal basis in separable Hilbert spaces.Fourier Series with respect to an orthonormal basis.

Unit-IV

Projection Theorem, Riesz Representation Theorem. counterexample to the Projection theorem and Riesz Representation Theorem for incomplete spaces, Hilbert property of the dual of a Hilbert space and counterexamples for incomplete inner product spaces, reflexivity of Hilbert space, adjoint of a Hilbert space operator, weak convergence and Bolzano-Weirstrass property in Hilbert spaces, normal and unitary operators, finite dimensional spectral theorem for normal operators.

Recommended books:

1. Introduction to Topology and Modern Analysis, G. F. Simons.
2. Functional Analysis, B.V.Limaya, Newage Internationals.
3. A First Course in Functional Analysis, C.Goffman G. Pedrick PHI.
4. Elements of Functional Analysis, L.A. Lusternick & V.J. Sobolov,
5. A Course in Functional Analysis, J.B. Conway Springer.

MTH-704 – Measure Theory-I

Unit-I

Semiring, algebra and σ-algebra, measure on semi-ring with examples, outer measure and measurable sets, collection of all measurable sets Λ is a σ-algebra, σ-additivity of outer measure on Λ. Caratheodary extension of an outer measure. Lebesgue measure on $\mathbb{R}$, intervals are Lebesgue measurable and Lebesgue measure of an interval is its length. Existence of a non-measurable set.

Unit-II

Measurable functions with examples and their characterizations, algebra of measurable functions. Almost everywhere property, Limit of a sequence of measurable functions. Limit superior and limit inferior of a sequence of measurable functions. Egorov's Theorem, characteristic function and its properties, simple and step fictions. Standard representation of a step function, Integral of a step function and its linearity. Monotonicity of the integral and the order continuity of the integral and related results.

Unit-III

Lebegue measure on $\mathbb{R}^n$, interval of $\mathbb{R}^n$. Criterion for a subset of $\mathbb{R}^n$ to be Lebesgue measurable. Borel set and Borel subsets of $\mathbb{R}^n$, regular Borel measure, Borel measure. The Lebesgue measure on $\mathbb{R}^n$ is regular Borel measure, translation invariance of Borel measure on $\mathbb{R}^n$. Steinhauss's theorem for $\mathbb{R}^n$. Ostroviski's therem on measurable solution of $f(x+y) = f(x) + f(y)$, where x, y are real numbers Convergence a.e, convergence in measure, almost uniform convergence and their relationship.

Unit-IV

Upper functions, generating sequence for upper functions, algebra of upper functions, integrable functions, the class of all integrable functions is a function space. Some theorems on integrable functions. Levi's theorem, Fatou's lemma. Lebesgue dominated convergence theorem.The Riemann integral as a Lebesgue integral, Lebesgue-Vitali theorem.

Recommended books:

1. Principles of Real Analysis, C. D. Aliprantis, O. Burkinshaw, Academic Press.
2. Real Analysis, P. M. Fitzpatrick and H. L Royden, Pearson.
3. An Introduction to Measure and Integration, I. K. Rana, Narosa

MTH-705 – Advanced Number Theory

Unit-I

Number-theoretic functions, multiplicative functions, totally multiplicative functions, basic examples. Functions: $\tau(n)$, $\sigma(n)$ and their properties. Mersenne numbers, Perfect numbers, necessary and sufficient condition for an even number to be perfect. Moebius function $\mu(n)$, Moebius inversion formula, sum of odd divisors of n, $\sum_{d|n}\mu(d)\tau(d)$, $\sum_{d|n}\mu(d)\sigma(d)$ and related results. Euler's constant γ, symbols o, O and $\sim$, average order of $\phi(n)$. Greatest integer function.

Unit-II

Order of an element modulo(m), related results, if $d|p-1$, the congruence $x^d \equiv (mod\ p)$ has exactly d solutions. Primitive roots, primitive roots of an odd prime, primitive root of an odd prime with any power. Universal exponent of $\lambda(m)$ of m, refinement of Euler's theorem, the numbers having primitive roots are $1, 2, 4, p^\alpha$ and $p^{2\alpha}$, where p is any odd prime. Quadratic residues and quadratic non-residues, Euler's criterion and related results.

Unit-III

Legendre symbols, its properties, lemma of Gauss, quadratic reciprocity law for Legendre symbols, characterization of primes for which $2, -2, 3, -3, 5$ are quadratic residue or quadratic non-residue. Jacobi symbols, properties, quadratic reciprocity law for Jacobi symbols and other related results. Farey fractions, Farey sequences, rational approximations, their related results.

Unit-IV

Simple finite continued fractions, infinite continued fractions, irrational numbers, infinite continued fraction is irrational and related results. Irrationality of e and π, Hurwitz theorem and its sharpness. Character, orthogonality relation for characters, Riemann zeta function $\zeta(s) = \prod_p \left(1 - \frac{1}{p^s}\right)^{-1}$ for $s > 1$, $\frac{1}{\zeta(s)} = \sum_{m=1}^{\infty} \frac{\mu(m)}{m^s}$. Algebraic numbers and Algebraic integers, Algebraic number field, Primes in Quadratic forms, Euclidean Quadratic forms, unique factorization, solution of $x^2 + y^2 = z^2$ and $x^3 + y^3 = z^3$ in rational integers.

Recommended books:

1. Elementary number Theory, David M. Burton, McGraw Hill Education.
2. An introduction to the Theory of Numbers, I. Niven and H.S. Zuckermann, Wiley.
3. Topics in Number Theory, W. J. Leveque, Dover Publications.

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Semester-VIII

Course Structure				Marks			
S.No.	**Course Code**	**Course Title**	**Credits**	**CIA**	**MT**	**ET**	**TOTAL**
1	MTH-801	Complex Analysis-II	4	20	30	50	100
2	MTH-802	Field and Galois Theory	4	20	30	50	100
3	MTH-803	Topology-II	4	20	30	50	100
4	MTH-804	Measure Theory-II	4	20	30	50	100
5	MTH-805	Partial Differential Equations	4	20	30	50	100
6	SO	Social Orientation (Open Elective)	4	20	30	50	100
Total			24	120	180	300	600

Semester VIII

MTH-801 – Complex Analysis-II

Unit-I

Maximum Modulus Principle, Schwarz Lemma and its generalization, Meromorphic functions, Argument Principle, Rouche's theorem with applications, Inverse function Theorem, Poisson integral formula for a circle and half plane, Poisson Jenson formula, Carleman's theorem, Hadamard three-circle theorem and the theorem of Borel and Caratheodory.

Unit-II

Principle of analytic continuation, uniqueness of direct analytical continuations and uniqueness of analytic continuation along a curve. Power series method of analytic continuation, Functions with natural boundaries and related examples. Schwartz reflection principle, functions with positive real part.

Unit-III

Space of analytic functions, Hurwitz's theorem, Montel's theorem, Riemann Mapping theorem, Weirstrass factorization theorem, Gamma function and its properties. Riemann Zeta function, Reimann's functional equation. Harmonic functions on a disc, Harnack's inequality and theorem, Dirichlet's problem.

Unit-IV

Canonical products, order of an entire functions, Exponential convergence, Borel theorem, Hadamard's factorization theorem, The Range of analytic functions, Bloch's Theorem, Schottky's Theorem, The Little Picard's Theorem, Landau's Theorem, Great Picard Theorem (statement and applications only), Univalent function. Bieberbach's conjecture (statement only) and the 1/4-theorem.

Recommended books:

1. Complex Analysis, L. Ahlfors, McGraw Hill Education.
2. Theory of Functions, E.C. Titchmarsh Oxford University Press.
3. Functions of one complex variable, J.B. Conway, Narosa.
4. Complex Analysis, Richard Silverman, Dover publications.
5. Theory of Functions of a Complex variable, A. I. Markushevish, AMS/Chelsea Publication.

MTH-802 – Field and Galois Theory

Unit-I

Field extensions: basic results and examples, degree of extension and related results, finite and algebraic extensions, finitely generated and simple extensions. Classical straightedge and compass constructions.

Unit-II

Splitting fields: examples and related results, existence and uniqueness of splitting fields, Algebraic closure and algebraically closed fields, separable and inseparable extensions, prefect fields. Characterization of finite fields, polynomials over finite fields, traces and norms. Cyclotomic polynomials and extensions.

Unit-III

Field automorphisms and fixed fields, Galois groups and Galois extensions. Fundamental Theorem of Galois Theory, with applications and related results. Composite extensions, cyclotomic extensions and abelian extensions over $\mathbb{Q}$.

Unit-IV

Symmetric and elementary symmetric functions. Discriminant of a polynomial, Galois groups of polynomials of degree 2, 3 and 4. Radical extensions, Solvability by radicals of cubic and quartic equations and Insolvability of the quintic.

Recommended books:

1. Abstract Algebra, D. S. Dumit and R. M. Foote, Wiley Sons.
2. Finite Fields, Lidl Rudolf, Cambridge University Press, 2nd Edition.
3. Field and Galois Theory, Morandi Patrick, Springer.
4. Galois Theory, Artin Emil, Dover Books on Mathematics.
5. Galois Theory, Ian Stewart, Chapman and Hall.

MTH-803 – Topology-II

Unit-I

Review of Compactness and finite intersection property. Product spaces, Tychnoff's theorem, Generalized Heine-Borel theorem. Limit point, compactness and local compactness, one point compactificationg. Separation Axioms: T_1 spaces and Hausdroff spaces, regular and normal spaces, Urysohn Lemma, Urysohn Metrization Theorem, Tietze Extension Theorem.

Unit-II

The function Algebras $\mathcal{C}(X, R)$ and $\mathcal{C}(X, C)$ and related results, equicontinous, Ascoli's theorem. The Stone-Cech compactification. Approximation: The Weierstrass approximation theorem, the real Stone Weiertstrass theorem and the complex Stone Weierstrass theorem, the extended Stone Weierstrass theorems.

Unit-III

Paths and path homotopy with examples, homotopy equivalence, contractibility, deformation retracts and examples. Basic constructions: cones, mapping cones, mapping cylinders, suspension.

Unit-IV

Fundamental groups: the fundamental group of the circle and its applications (including Fundamental Theorem of Algebra, Brouwer Fixed Point Theorem and Borsuk-Ulam Theorem). Van Kampen's Theorem, Covering spaces. Lifting of paths and homotopies, lifting properties.

Recommended books:

1. Algebraic Topology, A. Hatcher, Cambridge Univ. Press, Cambridge, 2002.
2. Introduction to Topology and Modern Analysis, G. F. Simmons, Tata McGraw Hill Education.
3. Topology, J. R. Munkres, Pearson.
4. General Topology, J. L. Kelley, Springer.
5. Algebraic topology: A First Course, W. Fulton, Springer-Verlag, 1995.
6. A Basic Course in Algebraic Topology, W. Massey, Springer-Verlag, Berlin, 1991.
7. An Introduction to Algebraic Topology, J. J. Rotman, Springer (India), 2004.
8. Elements of Algebraic Topology, J. R. Munkres, Addison-Wesley, 1984.

MTH-804 – Measure Theory-II

Unit-I

Continuity of monotone functions, differentiability of monotone functions, Vitali cover, Vitali covering lemma, Lebesgue's theorem, divided difference functions and average value functions, functions of bounded variation, Jordan's theorem.

Unit-II

Absolutely continous functions, Cantor-Lebesgue function, relationship between bounded variation functions and absolutely continuous functions, uniformly integrable and absolutely continuous functions, absolutely continuous functions and indefinite integral, fundamental theorem of calculus.

Unit-III

L^p spaces, Young's inequality, Hölders inequality, Minkowski's inequality, Cauchy-Schwarz's inequality and their applications. Riesz-Fischer theorem,completion of $C[a, b]$. Product measure, Iterated integrals, Fubbibi's theorem and Tonells's theorem.

Unit-IV

Collection of all measures on $\sum$ is a lattice, signed measure, Hahn decomposition with respect to signed measure, the collection of all finite signed measures on a σ-algebra is Banach lattice, comparing measure and Radon-Nikodym theorem.

Recommended books:

1. Principles of Real Analysis, C. D. Aliprantis, O. Burkinshaw, Academic Press.
2. Real Analysis, H. L. Royden, P. M. Fitz Patrick, Pearson.
3. Real and Complex Analysis, Walter Rudin, McGraw Hill Education.
4. An Introduction to Measure and Integration, I. K. Rana, Narosa.

MTH-805 – Partial Differential Equations

Unit-I

Introduction to Partial Differential Equations (PDEs), Classification of PDEs of first order, Formation of PDEs, Linear PDEs, Lagrange's method for the solution of first order PDEs, Integral surfaces passing through a given curve, Non-linear PDEs, Compatible system of first oder non-linear PDEs, Charpit's method for the solution of non-linear PDEs, Cauchy's problem for non-linear PDEs.

Unit-II

Linear PDEs of second order with constant coefficients and their solutions, characteristic roots and characteristic curves for second order PDEs, Canonical forms of second order PDEs, Monge's method for solving second order non-linear PDEs, Derivation of Heat, Wave and Laplace's equation and their solution by variables separable method.

Unit-III

D'Alembert's solution of Wave equation with finite, infinite string length, Duhamel's principle for solving inhomogeneous wave equation, Poisson method of spherical averages, Hadamard's method of descent, Mean value property and Max-Min principle, Green's function for half plane and disc and fundamental solution, Wave equation in higher dimension, Method of spherical means, Kirchoff's formlula, Huygen's principle.

Unit-IV

Fourier transform: definition, L^1-theory, Reimann-Lebesgue theorem, L^2-theory, Plancheral theorem, L^p- theory, Dirichlet's principle and uniqueness theorem for Laplace equation.

Recommended books:

1. Partial Differential Equations for Engineers and Scientists, J.N. Sharma and Kehar Singh, Narosa Publications.
2. An Elementary Course in Partial Differential Equations, T. Amarnath, Narosa Publictions.
3. Beginning Partial Differential Equations, Peter V.O'Neil, Wiley Edition.
4. Nonlinear Partial Differential Equations for Scientists and Engineers, Lokenath Debnath, Birkhäuser.
5. Partial Differential Equations for Scientists and Engineers, Stanley J. Farlow, Dover Publications Inc.

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Semester-IX

Course Structure				Marks			
S.No.	**Course Code**	**Course Title**	**Credits**	**Marks**			**TOTAL**
1	MTH-901	Project Work	4	20†	30††	50†††	100
S.No.	**Course Code**	**Course Title**	**Credits**	**CIA**	**MT**	**ET**	**TOTAL**
2	OGE	Open Generic Elective	4	20	30	50	100
		Any Four of the Following					
3	MTH-E902	Theory of Semigroups-I	4	20	30	50	100
4	MTH-E903	Commutative Algebra	4	20	30	50	100
5	MTH-E904	Analytic Theory of Polynomials	4	20	30	50	100
6	MTH-E905	Category Theory	4	20	30	50	100
7	MTH-E906	Advanced Topics in Graph Theory	4	20	30	50	100
8	MTH-E907	Differentiable Manifolds	4	20	30	50	100
9	MTH-E908	Optimization Techniques and Control Theory	4	20	30	50	100
10	MTH-E909	Inferential Statistics	4	20	30	50	100
11	MTH-E910	Calculus of Variations and Integral Equations	4	20	30	50	100
12	MTH-E911	Algebraic Cryptography	4	20	30	50	100
13	MTH-E912	Frames and Wavelets	4	20	30	50	100
14	MTH-E913	Lie Algebras	4	20	30	50	100
		Total	24	100	150	250	500
		Grand Total					600

† Literature Review and Review Writing, †† Seminar Presentation, ††† Written Exam.

Semester IX

MTH-901 – Project Course Work

† **Literature Review and Review Writing** The students shall be asked to review some literature and write a review of at least one research article related to their project.

†† **Seminar Presentation** The students shall have to make seminar presentations as per the schedule prepared by their concerned supervisor. There shall be one presentation at the departmental level as well.

††† **Written Exam** There shall be an internal presubmission written exam under the supervision of concerned guide and head of the department. The question paper will be of 50 marks. The pattern of the question paper shall be uniform for all the groups.

Guide/Supervisor Eligibility: The Guide/Supervisor must possess atleast one research paper published in SCI/SCOPUS indexed journal.

Recommended books:

- Books shall be recommended by the concerned guide/supervisor, who will conduct the examination of the course, accordingly under the supervision of the HOD.

MTH-E902 – Theory of Semigroups-I

Unit-I

Basic definitions and examples of semigroups. Direct product of semigroups, rectangular bands. Monogenic semigroups, ordered sets, semilattices and lattices. Binary relations and partial mappings, equivalences and congruences. Homomorphism Theorems. Generating equivalences and congruences.

Unit-II

Free semigroups and free monoids, semigroup presentations, ideals and Rees congruences, lattices of equivalences and congruences. Green's equivalences: Definitions, examples and related results. Green's lemma and related results.

Unit-III

Regular D-classes and related results, regular semigroups and related results, Lallement's Lemma. Simple and 0-simple semigroups, minimal and 0-minimal ideals. Inverse semigroups, symmetric inverse semigroups, Vagner-Preston Theorem..

Unit-IV

Systems, bi-systems and subsystems, tensor product of systems; its existence and uniqueness, some properties of tensor product. Free product of semigroups, free product as a coproduct. Semigroup amalgams and its embaddability, amalgamated free product of semigroups. Dominions and zigzags and their relation with semigroup amalgams, Isbell's Zigzag Theorem on dominions and its applications.

Recommended books:

1. Fudamentals of Semigroup Theory, J. M. Howie, Oxford Science Publications.
2. The Algebraic Theory of Semigroups, A. H. Clifford and G. B. Preston, American Mathematical Scociety.
3. Nine Chapters on the Semigroup Art, Alan J. Cain, Porto & Lisbon.

MTH-E903 – Commutative Algebra

Unit-I

Rings, ring homomorphism, ideals, quotients, zero divisors, nilpotents and units. Prime and maximal and comaximal ideals, nilradical, Jacobsons radical and radical of an ideal. Operations on ideals, extension and contraction. Chinese remainder theorem and prime avoidance lemma.

Unit-II

Review of modules, sub-modules, module homomorphisms. Finitely generated modules, Nakayama lemma, exact sequences, split exact sequences, four lemma, five lemma, short five lemma, free, projective and injective modules, Tensor product of modules, flat modules. Restriction and extension of scalars. Rings and modules of fractions and localization.

Unit-III

Chain conditions, Noetherian and Artinian rings. Hilbert basis theorem, primary decomposition in Notherian rings and Artinian rings and structure theorem on Artinian rings, modules over PID. Modules of finite length and primary Decomposition.

Unit-IV

Integral dependence, Going-up and Going-down theorems, valuation rings, integral extensions, Hilbert's Nullstellansatz, Noether normalization theorem.

Recommended books:

1. An Introduction to Commutative Algebra, M. F. Atiyah and I. G. Macdonald, Addison-Wesley.
2. Introduction to Commutative Algebra and Algebraic Geometry, E. Kunz, Birkhäuser.
3. Cohen-Macaulay Rings, W. Bruns and J. Herzog, Cambridge University Press.
4. Commutative Ring Theory, H. Matsumura, Cambridge University Press.

MTH-E904 – Analytic Theory of Polynomials

Unit-I

The fundamental theorem of algebra (revisited). Symmetric polynomials. The Continuity theorem. m-Distribution and the associated system of orthogonal polynomials: general properties and three-point recurrence formula. Christoffel-Darboux and Gaussian quadrature formulae. Trigonometric and classical orthogonal polynomials. Tools from Matrix Analysis: Companion matrix, Hardmard's theorem, Greschgorin discs and Greschgorin disc Theorem.

Unit-II

Critical points of a polynomial and their relationship with the zeros: critical points as convex linear combination of zeros of a polynomial. Convex Hulls and Gauss-Lucas theorem. An Extension of Gauss-Lucas theorem: Kuiper's Theorem. Average distance from a line or a point. Real polynomials and Jenson's theorem. Extension of Jenson's theorem.

Unit-III

Derivative estimates on the unit disc: Bernstein's inequality, some of its generalizations and refinements due to Rahman and Schmeisser and A. Aziz. Derivative estimates for polynomials with restricted zeros: inequalities due to Erdös -Lax and Paul Turàn. Derivative estimates on the unit interval: Bernstein's pointwise estimate and inequality due to A. Markov. Extensions to higher order derivatives. Result due to Schur.

Unit-IV

Rational functions and some inequalities for the maximum modulus of the derivative of a rational function with prescribed poles: extension of inequalities due to Bernstein, Erdös-Lax and Paul Turàn. Derivative estimate for functions of exponential type on the real line: analogue of Bernstein's inequality for functions of exponential type. L_p analogues of some polynomial inequalities: inequalities due to Zygmund and De Bruijn.

Recommended books:

1. Analytic theory of Polynomials, Q.I. Rahman and G. Schmeisser, Clarendon Press.
2. Geometry of polynomials, Morris Marden, American Mathematical Society.
3. Topics in polynomials: extremal properties, problems, inequalities, zeros, G.V. Milovanovic, D.S. Mitrinovic and Th. M. Rassias, World Scientific.
4. Problems and theorems in Analysis II, G. Polya and G. Szego, Springer

Suggested Papers:

1. Bernstein-Type Inequalities for Rational Functions with Prescribed Poles, Xin Li, R. N. Mohapatra And R. S. Rodriguez, J. London Math. Soc. (2) 51 (1995) 523-531.

MTH-E905 – Category Theory

Unit-I

Categories: definitions, examples and basic properties, isomorphisms. Construction on categories: opposite categories and duality principle, product of categories, arrow category, slice category, large, small and locally small categories. Functors: examples and basic properties. Covariant and contra-variant functors. Full, faithful, amnestic functors and embeddings. Equivalences and equivalent categories.

Unit-II

Subcategories, full subcategories examples and properties. Fully embeddable categories, isomorphism, closed categories and skeleton, Monoidal categories. Reflections, co-reflections, reflective and co-reflective subcategories and related results. Natural transformations and natural isomorphisms. Representable functors and yoneda lemma.

Unit-III

Initial and terminal objects, zero objects. Separators and co-separators, sections and retractions. Monomorphisms and epimorphisms, bimorphisms and balanced categories. Equalizers and co-equalizers.

Unit-IV

Regular and extremal monomorphisms and epimorphisms. Pullbacks and pushouts. Products and coproducts. Limits and co-limits. Complete and co-complete categories.

Recommended books:

1. Abstract and concrete categories, J. Adamck, H. Herlich, and G. Strecuer, Dover Books on Mathematics.
2. Handbook of categorical Algebra-I, F. Borcuex, Cambridge University Press.
3. Category Theory, S. Awodey, Oxford University Press.

MTH-E906 – Advanced Topics in Graph Theory

Unit-I: Coloring of Graphs
Vertex coloring, Brook's theorem, Nordhaus-Gaddum Theorem, Edge colouring of graphs, Konig's theorem, Vizing's theorem, Region coloring graphs, five color theorem, Heawood map colouring theorem, Tait's theorem, uniquely colourable graphs. Chromatic polynomial of complete graph, totally disconnected graph, cycle, tree. Chromatic polynomial by identification of vertices, Hassler Whitney's Theorem.

Unit-II: Matchings and Factorization
Matchings, Berge's theorem, Hall's condition, The marriage theorem, perfect matchings, Factors, Tutte's 1−factor theorem, perfect matchings in regular bipartite graphs, Peterson's theorem, factorisation of complete graphs.

Unit-III: Edge Graphs and Digraphs
Edge graph, Krausz theorem, Beineke's theorem, edge graphs of trees and Eulerian property of edge graphs. Digraphs, strong, weak and unilateral digraphs, Euler digraphs, Matrices of digraphs: Incidence matrix, cycle matrix, Cut-set matrix, adjacency matrix, Tournaments, Redei's theorem, Camion's theorems.

Unit-IV: Algebraic Graph Theory: An Introduction
Automorphism groups of graphs, Cayley group of a graph, Spectrum of some graphs − complete graph, complete bipartite, cycle and path, regular graph, compliment of a graph, edge graph. Determinant of adjacency matrix and Sach's theorem, Laplacian matrix, rank of Laplacian matrix, Laplacian spectrum of regular graphs, relation between the Laplacian spectrum of graph and its complement.

Recommended books:

1. A Text Book of Graph Theory, R. Balakrishnan, Ranganathan, Springer- Verlag.
2. Extremal Graph Theory, B. Bollobas, Academic Press.
3. Chromatic graph theory, Gary Chartrand, Ping Zank, CRC Press Taylor and Francis group.
4. An Introduction to Graph Theory, S. Pirzada, Universities Press, Orient Blackswan, Hyderabad, India, (2012).
5. Graph Theory, F. Harary, Addison-Wesley.
6. Graph Theory with Applications to Engineering and Computer Science, Narsing Deo, Prentice Hall.
7. A First book at Graph Theory, J. Clark and D. A Holton, World Scientific.
8. D. B. West, Introduction to Graph Theory, Prentice Hall.

MTH-E907 – Differentiable Manifolds

Unit-I

Calculus of $\mathbb{R}^n$: Continuity and differentiability of functions from $\mathbb{R}^n \to \mathbb{R}^n$, Chain rule, Differential map, Inverse mapping theorem and its implications in geometry, diffeomorphisms.

Unit-II

Manifolds: Topological manifold, Atlas, smooth Manifold, Examples of manifolds, Differentiable structure on a manifold, Space of smooth maps, Differential of a smooth map

Unit-III

Submanifolds: Immersion, Embedding and submanifolds with examples, tangent space of submanifolds, Vector fields and smooth maps, Lie brackets, Vector bundles, Cotangent bundle, tangent covectors on manfiolds.

Unit-IV

Differential forms and connections: Tensors, Contravariant and covariant tensors, Mixed tensors, Alternating tensors, Differential forms, Pull back, Differential forms on manifolds, Exterior derivative.

Recommended books:

1. A course in Differential Geometry and Lie groups, S. Kumaresan, Hindustan Book Agency.
2. An introduction to Differential Manifolds and Riemannian Geometry, W. M. Boothby, Academic Press; 2nd edition.
3. J. M. Lee: Introduction to Smooth Manifolds, Graduate Texts in Mathematics, Springer.
4. M. Spivak: A comprehensive introduction to differential geometry, Publish or Perish, Inc., Houston, Texas, 2005.

MTH-E908 – Optimization Techniques and Control Theory

Unit-I

Extended real valued functions, Proper convex functions, Subgradients, Directional derivatives.

Unit-II

Conjugate functions, Dual convex programs, Optimality conditions and Lagrange multipliers, Duality and optimality for standard convex programs, Gradient descent method, Gradient projection method.

Unit-III

Newton's method, Conjugate gradient method, Dynamic programming, Bellman's principle of optimality, Allocation problem, Stage coach problem.

Unit-IV

Optimal control problem and formulations, Variational approach to the fixed-time free endpoint problem, Pontryagin's maximum principle, Dynamic programming and Hamilton-Jacobi-Bellman equation.

Recommended books:

1. Nonlinear Programming: Analysis & Methods, M. Avriel, Dover Publications, New York, 2003.
2. Foundations of Optimization, O. Güler, Springer 2010.
3. Introduction to Operations Research, F.S. Hillier, G.J. Lieberman, P. Nag and P. Basu, Tata McGraw-Hill, 2012.
4. Calculus of Variations and Optimal Control Theory: A Concise Introduction, D. Liberzon, Princeton University Press, 2012.

MTH-E909 – Inferential Statistics

Unit-I

Sampling theory: random sample and statistic. Distributions of functions of random variables: change of variables method, distributions of order statistics, transformations of variables of the discrete type, transformations of the variables of the continuous type, extensions of the change of variable technique, Moment Generating function technique, the distribution Function method. The Limiting distribution, Stochastic convergence, limiting moment generating functions, the central limit theorem, some theorems on limiting distributions (statements only).

Unit-II

Interval estimation and random intervals, confidence intervals for means, confidence intervals for difference of means, confidence interval for variances. Point estimation, A sufficient statistic for a parameter, Fisher Neyman Criterion, Factorization theorem, Rao-Blackwell theorem, Completeness and uniqueness, Complete sufficient statistic, Best statistic, the exponential class of probability density functions.

Unit-III

Further topics in point estimation, The Rao-Crammer inequality, Efficient estimators, Consistent estimators, Maximum likelihood estimation of parameters. Relation between maximum likelihood estimators and sufficient estimators.

Unit-IV

Testing of Hypothesis: Concepts of critical regions, test functions, types of errors, size function, power function, level of significance, Most Powerful (MP) and Uniformly most powerful (UMP) test. Neyman Pearson Lemma, MP tests for simple null against simple alternative hypothesis. Chi-square tests.

Recommended books:

1. Vijay K. Rohatgi and A.K.Md. Ehsanes Saleh, An Introduction to Probability and Statistics, John Wiley & Sons.

2. Robert V. Hogg, Joseph McKean and Allen T. Craig, Introduction to Mathematical Statistics, Pearson Education Inc.

3. S.C. Gupta and V.K. Kapoor, Fundamentals of Mathematical Statistics, Sultan Chand & Sons.

MTH-E910 – Calculus of Variations and Integral Equations

Unit-I

Calculus of Variations: Introduction, differentiability and the first Variation of a Functional. Necessary Condition for Extrema. The Fundamental Lemma of the Calculus of Variations and the Euler-Lagrange Equation. Some Typical Problems of The Subject-The Brachistochrone Problem. Lagrange Multipliers and Isoperimetric Problems.

Unit-II

Variational Problems with Moving (or Free) Boundaries. The Second Variation and Sufficient Condition for Extremum of a Functional. Approximate Solutions of Boundary Value Problems by Rayleigh-Ritz Method.

Unit-III

Linear Integral Equations: Introduction, Linear Integral Equation of The First and Second Kind of Fredholm And Volterra Type. Solution of Fredholm Integral Equations of the Second Kind with Separable Kernels. Solution of Volterra Integral Equations of First and Second Kind.

Unit-IV

Orthonormal System of Functions, Eigen Values and Eigen Functions. Hilbert-Schmidt Method of Solving Non-Homogeneous Fredholm Integral Equations of Second Kind.

Recommended books:

1. Calculus of variation, M. Gelfand and S. V. Fomin, Dover.
2. Calculus of variations with applications to physics, R. Weinstocks, Dover.
3. Integral equations, F. G. Tricomi, Dover.
4. Introduction to nonlinear differential equations and integral equations, H. L. Davis, Dover.

MTH-E911 – Algebraic Cryptography

Unit-I

Introduction to modern cryptography and its principles, data encryption and data decryption, Symmetric (private) and Asymmetric (public) cryptography, Some well known algorithms: AES, DES, IDES, and Random number generator (RNG, Classical cryptography, simpe substitution cipher, vigenere cipher,caesar cipher, Hill cipher and affine Hill cipher, cryptanalysis and brute force attack.

Unit-II

Introduction to asymmetric cipher, origin of public key cryptography, the discrete logarithm problem, Diffie-Hellman key exchange algorithm, the Elgamal public key cryptosystem, hardness of discrete logarithm problem.

Unit-III

Integer factorization and RSA cryptosystem, implementation and security issues of RSA cryptosystem, Euler's and Fermat's formula, primality testing, Miller-Rabin test for composite numbers, introduction and implementations of elliptic curve cryptography, hash function and its applications.

Unit-IV

Digital signatures: components of a digital signature scheme, RSA digital signature, Elgamal digital signature, digital signature algorithm (DSA), Goldreich-Goldwasser-Halevi (GGH) lattice based digital signature scheme, NTRU digital signature scheme, stream cipher vs block cipher, one time pad, linear feedback shift register.

Recommended books:

1. An introduction to mathematical cryptography, J. Hoffstein, J. Pipher, J. Silverman, Springer.
2. Cryptography and network security, William stalling, Pearson.
3. Understanding cryptography, C. paar, J. Pelzl, Springer.

MTH-E912 – Wavelets and Frames

Unit-I

Definition and examples of Fourier Transform in $L^2(\mathbb{R})$, basic properties of Fourier transforms, Plancherel's and Parseva'ls formlae, energy preserving relation, Convolution and Correlation, Poisson summation formula, Shannon-Whittaker, sampling theorem, Heisenberg's uncertainty principle.

Unit-II

Motivation and definition of continuous wavelet transforms (CWT), basic properties of wavelet transforms, Haar wavelet, Maxican hat wavelet, Meyer wavelet and their Fourier transforms, Parseval's, formula and energy preserving relation of CWT, reconstruction formula for CWT. Discrete wavelet transform and applications.

Unit-III

Finite frames, Canonical reconstruction formula, Frames and matrices, Similarity and unitary equivalence of frames, Frame bounds and frame algorithms.

Unit-IV

Frames and Bessel sequences in infinite dimensional Hilbert spaces, Frame sequence, Gram matrix, Frames and operators, Characterization of frames, Dual frames, Tight frames, Continuous frames, Frames and signal processing, Tight frames and dual frame pairs.

Recommended books:

1. I. Daubechies, Ten Lectures on Wavelets, SIAM, Philadelphia, 1992.
2. D.K Ruch and P.J. Van Fleet, Wavelet Theory, John Wiley, 2009.
3. E. Hernandez and G. Weiss, A First Course on Wavelets, CRC Press, New York, 1996.
4. D. Han, K. Kornelson, D. Larson and E. Weber, Frames for Undergraduates, American Mathematical Society, Student Mathematical Library, Volume 40, 2007.
5. O. Christensen, An Introduction to Frames and Riesz Bases, Second Edition, Birkhäuser.

MTH-E913 – Lie Algebras

Unit-I

Definition and examples of Lie algebras, classical Lie algebras, derivations of Lie algebras, abelian Lie algebras, Lie subalgebras, ideals and homomorphisms, normalizers and centralizers of a Lie subalgebra, representations of Lie algebras (definition and examples), automorphisms of a Lie algebra.

Unit-II

Solvable algebra, Solvable radical, nilpotent algebra, semi simple Lie algebra, Lie's theorem, Jordan-Chavalley decomposition (exsitence and uniqueness), Cartan trace criterion for solvalbility, Killing form and criterion for semi simplicity.

Unit-III

Simple ideals, inner derivations, abstract Jordan-Chavalley decomposition, Lie algebra modules, Schur's lemma, Casimir elements of a representation, Weyl's theorem for preservation of Jordan-decomposition.

Unit-IV

Representation of $SL(2, \mathbb{C})$: weights, highest weight, maximal vectors, classification of irredicible modules, toral and maximal toral subalgebra, root space decomposition and properties of roots.

Recommended books:

1. J. E. Humphreys, Introduction to Lie algebras and Representation Theory, Graduate Text in Mathematics, (, Springer-Verlag, 1980..
2. N. Jacobson, Lie Algebras, Wiley-Interscience, New York, 1962.
3. J. P. Serre, Lie Algebra and Lie Groups, Benjamin, New York, 1965.

Integrated BSc – MSc Mathematics
Revised Syllabus

Semester-X

Course Structure				Marks				
S.No.	**Course Code**	**Course Title**	**Credits**	**Marks**				**TOTAL**
1	MTH-1001	Project Dissertation	8	Dissertation* 100	Viva 100†			200
Any Four of the Following								
S.No.	**Course Code**	**Course Title**	**Credits**	**CIA**		**MT**	**ET**	**TOTAL**
2	MTH-E1002	Theory of Semigroups-II	4	20		30	50	100
3	MTH-E1003	Functional Analysis-II	4	20		30	50	100
4	MTH-E1004	Universal Algebra	4	20		30	50	100
5	MTH-E1005	Representation Theory	4	20		30	50	100
6	MTH-E1006	Algebraic Geometry	4	20		30	50	100
7	MTH-E1007	Spectral Graph Theory	4	20		30	50	100
8	MTH-E1008	Complex dynamics	4	20		30	50	100
9	MTH-E1009	Algebraic Number Theory	4	20		30	50	100
10	MTH-E1010	Riemannian Geometry	4	20		30	50	100
11	MTH-E1011	Topics in Non-Commutative Algebra	4	20		30	50	100
12	MTH-E1012	Special Functions	4	20		30	50	100
13	MTH-E1013	Non-Linear Analysis	4	20		30	50	100
14	MTH-E1014	Harminic mappings in a plane and Univalent functions	4	20		30	50	100
Total			24	80		120	200	400
Grand Total								600

∗ External, (†) 20 Internal, 80 External

Semester X

MTH-1001 – Project Dissertation

Project-Dissertation

Project dissertation would consists of five chapters based on the survey of at least two research articles published in SCI/SCOPUS indexed journals. The dissertation shall be evaluated by an external expert and has to be defended by the student in an open viva-voce. The external expert shall also be part of the committee for the conduct of viva-voce.

Recommended books:

- Books shall be recommended by the concerned Guide/Supervisor, who will conduct the examination of the course, accordingly.

MTH-E1002 – Theory of Semigroups-II

Unit-I

Compeletely 0-simple semigroups, Regular Rees Matrix semigroups, Rees Theorem on completely 0-simple semigroups and related results. Isomorphism and normalization of completely 0-simple semigroups, congruences on completely 0-simple semigroups.

Unit-II

Completely regular semigroups, completely regular semigroups as semilattices of completely simple semigroups. Clifford semigroups as strong semilattices of groups, isomorphism of clifford semigroups. Review of inverse semigroups, the natural order relation on inverse semigroups. Brandt semigroups, definitions, examples and related results.

Unit-III

Congruences on inverse semigroups, definitions, examples and basic properties, kernel, trace congruence pairs, normal congruences on inverse semigroups, idempotent separating congruence, group congruences, maximum and minimum idempotent separating congruence.

Unit-IV

Varities of $(2, 1)$ algebras, equational classes and varieties, examples of some well known varieties of semigroups. Varieties of bands. Structure Theorems on some varieties of bands.

Recommended books:

1. Inverse Semigroups, Mario Petrich, John Wiley and Sons.
2. Fundamentals of Semigroup Theory, J. M. Howie, Oxford University Press, 1994.
3. Nine Chapters on the Semigroup Art, Alan J. Cain, Lecture Notes Porto, 2013.

MTH-E1003 – Functional Analysis-II

Unit-I

Weak and weak* topologies on a Banach space, Dual of a Banach space in its weak* topology, Goldstine's theorem, Banach-Alaoglu theorem and its simple consequences, Reflexivity of Banach spaces and weak compactness, Geometric and separation forms of Hahn-Banach Theorem, Applications of Hahn-Banach Theorem to: Banach limits, Markov-Kakutani theorem for a commuting family of maps, Dual of Subspace, Quotient space of a normed space.

Unit-II

Complemented subspaces of Banach spaces, Dixmier's theorem on the complement- tability of the dual of a Banach space in its bidual, uncomplementability of C_0 in ℓ_∞ and its consequences, Banachs closed range theorem, characterizations of injective and surjective bounded linear mappings between Banach spaces.

Unit-III

Completeness of $L_p[a, b]$, Duals of ℓ_∞ and L_p spaces, Mazur-Ulam theorem on isometries between real normed spaces, applications of fundamental theorems of functional analysis to Radon-Nikodym Theorem, non-surjectivity of Laplace transform and Muntz theorem for $C[0, 1]$.

Unit-IV

ℓ_∞ and $C[0, 1]$ as universal separable Banach spaces, ℓ_1 as a quotient universal separable Banach space. Extreme points, Krein-Milman theorem and its simple consequences, Banach Stone Theorem on isometries between $C(X)$ and $C(Y)$.

Recommended books:

1. Functional Analysis, B. V. Limaye, New Age International Publishers.
2. Infinite Dimensional Analysis, C. D. Aliprantis and K C Border, Springer.
3. A first course in functional Analysis, C. Goffman and G. Pedrick, Prentice Hall.
4. A Course in Functional Analysis, J.B. Conway, Springer Verlag.
5. Introduction to Banach Spaces and their geometry, B. Beauzamy, North Holland, Elsevier Science Ltd.
6. Functional Analysis, W. Rudin, Tata McGraw Hill.
7. An Introduction to Banach Space Theory, R. E. Megginson, Springer Verlag.
8. Linear Analysis, Ballobas. B, Camb. Univ. Press.

MTH-E1004 – Universal Algebra

Unit-I

Algebras, subalgebras,homomorphisms and direct products. Generating subalgebras, congruences and quotient algebras. Fundamental Homomorphism theorem. Ordered sets and lattices, Distributive, modular and complete lattices.

Unit-II

Closure operators and algebraic lattices, Galois connections. Ideals in lattices and related results. Isomorphism theorems. Direct products, subdirect products and subdirectly irreducible algebras.

Unit-III

Class operators and varieties. Terms, Term algebras and free algebras. Identities, equational classes, Birshoff's Theorem on varieties and related results.

Unit-IV

Fully invariant congruences and equational Theories. Congruence permutable variaties, congruence distributive, varieties, Matsev varieties, congruence modular varieties and arithmetical varieties.

Recommended books:

1. Universal Algebra, Clifford Bergman. CRC Press Taylor and Francis Group.
2. A course in Universal Algebra, Stannley Burris, H. P. Sankappanavar. Graduate Texts in Mathematics Springer Verlag.

MTH-E1005 – Representation Theory

Unit-I

Definitions, Basic examples, sub representations, irreducible representations, Subrepresentations, Tensor products of two representations, Symmetric and alternating Squares.

Unit-II

Character of a representation, Schur's lemma, Orthogonality relations, Decomposition of regular representation, Number of irreducible representations.

Unit-III

Canonical decomposition and explicit decompositions: Subgroups, Product groups, Abelian groups. Induced representations. Examples: Cyclic groups, alternating and symmetric groups.

Unit-IV

Integrality properties of characters, Burnside's pq theorem. The character of induced representation, Frobenius Reciprocity Theorem, Meckey's irreducibility criterion, Examples of induced representations.

Recommended books:

1. Representation Theory of Finite Groups, M. Burrow, Dover Books on Mathematics.
2. Linear Representation of Groups, J. P. Serre, Springer.
3. Algebra, S. Lang, Springer.
4. Representation Theory: A First Course, W. Fulton and J. Harris, Springer.

MTH-E1006 – Algebraic Geometry

Unit-I

Affine and projective varieties: Affine varieties, affine algebraic sets, Zariski topology, Hilbert's Nullstellensatz, Hilbert basis theorem, irreducible components. Projective varieties: projective algebraic sets, Zariski topology - projective varieties. Affine cones and projective Nullstellensatz.

Unit-II

Functions and morphisms: Regular and rational functions, regular functions on affine varieties, regular functions on quasi projective varieties. Morphisms: Definition of a morphism, morphisms to subvarieties of affine n-space, morphisms to quasi projective varieties.

Unit-III

Products: Products of affine varieties, The Segre Embedding, products of quasi projective varieties, completeness. Dimension and non-singularity: Dimension, finite morphisms.

Unit-IV

Tangents space and non-singularity, tangent cone of an affine variety, the tangent space of an affine algebraic set, tangent space for general varieties, rational maps and blowups, blowups of affine varieties, blowups of projective varieties.

Recommended books:

1. Algebraic Geometry, Robin Hartshorne, Graduate Texts in Mathematics GTM52, Springer.
2. The Red Book of Varieties and Schemes, David Mumford.
3. Algebraic Geometry: a first Course, Joe Harris, Springer-Verlag, New York (1992).
4. An Introduction to Commutative Algebra, M. F. Atiyah and I. G. Macdonald,Addison-Wesley.
5. Algebraic Curves: an Introduction to algebraic geometry, William Fulton, London (1969).

MTH-E1007 – Spectral Graph Theory

Unit-I

Matrices associated to a graph, Adjacency, Laplacian and Seidal matrices. Characteristic polynomial. The $(A, L, S)-$spectrum of graphs: Complete graph, Complete bipartite graphs, cycle, path, line graphs, regular graphs and complements.

Unit-II

Algebraic connectivity of graphs and bounds. Co-spectral graphs. $(A, L, S)-$spectrum of graphs of order up to 4. Spectral radius of graphs, Relations between spectral radius and chromatic number of graphs, Wifl's theorem.

Unit-III

Threshold graphs: Laplacian eigenvalues of threshold graphs, Equitable partition of graphs and divisor matrix. Relation between eigenvalues of a graph and its divisor matrix. Main spectrum of a graph, relation between the main spectrum and divisor matrix. Graphs with exactly one main eigenvalue.

Unit-IV

Adjacency spectrum of zero-divisor graphs of rings, Laplacian integrality of zero divisor graphs of rings of integer modulo.

Recommended books:

1. R. B. Bapat, Graphs and Matrices, Second Edition Springer, Universitext.
2. Bogdan Naca, A Brief Introduction to Spectral Graph Theory, EMS Text books in Mathematics.
3. Andries E.Brouwer and William H. Haemers, Spectra of Graphs, Universitext, Springer.
4. D. Cvetkovic, Peter Rowlenson, Slobodan Simic, An Introduction to Theory of Graph Spectra, London Mathemitcal Society.
5. Bolian Liu and Hong-Jain Lai, Matrices in Combinatorics and Graph Theory, Kluwer Academic Publishers.
6. Fuzhen Zhang, Matrix Theory, Basic results and Techniques, Universitext-Springer.

MTH-E1008 – Complex Dynamics

Unit-I

Iteration of a Mobius transformation, attracting, repelling and indifferent fixed points. Iterations of $R(z) = z^2, z^2 + c$. The extended complex plane, chordal metric, spherical metric, rational maps, Lipschitz condition, conjugacy classes of rational maps.

Unit-II

Valency of a function, fixed points, Critical points, Riemann Hurwitz relation. Equicontinuous functions, normality sets, Fatou sets and Julia sets completely invariant sets, Normal families and equicontinuity.

Unit-III

Properties of Julia sets, exceptional points backward orbit, minimal property of Julia sets. Julia sets of commuting rational functions, structure of Fatou set, Topology of the Sphere, Completely invariant components of the Fatou set.

Unit-IV

The Euler characteristic, Riemann Hurwitz formula for covering maps, maps between components of the Fatou sets, the number of components of Fatou sets, components of Julia sets.

Recommended books:

1. Iteration of rational functions, A. F. Beardon, Springer.
2. Complex dynamics, L. Carleson and T. W. Gamelin, , Springer.
3. Holomorphic dynamics, S. Morosawa, Y. Nishimura, M. taniguchi, T. Ueda, Cambridge University Press.
4. Dynamics of transcendental functions, X. H. Hua,C. C. Yang, Gordan and Breach Science Pub.
5. Dynamics in one complex Variable, John Milnor, Annals of Mathematics Studies, Princeton University Press.

MTH-E1009 – Algebraic Number Theory

Unit-I

Rings of integers, Dedekind Domains, Unique factorization of ideals, the ideal class group, Discrete valuations, integral closure of Dedekind Domains, factorization in extensions, examples.

Unit-II

Algebraic number fields and their conjugates, Algebraic integers, norm, Different and Discriminant of a number field, Integral Basis of an algebraic number field, multiplication and divisibility of ideals.

Unit-III

Norm of an integral domain, Norm and trace of an element, Norm of product of ideals and fractional ideals, Norm of a prime ideal, embedding, Dirichlet's Unit theorem, Fundamental system of units, S-units.

Unit-IV

Factoring primes in quadratic field, Monogenic number field, cubic field,arbitrary number field, Cyclotomic field, units in real quadratic fields, units of norm, 1, & -1, Absolute values, Non-Archimedean absolute values, equivalent absolute values.

Recommended books:

1. Introductory Algebraic Number Theory, Saban Alaca and Kenneth S. Williams, Cambridge.
2. Algebraic Number Theory, J. S. Milne, Springer.
3. Algebraic Number Theory, Robert B. Ash, Dover Publications.
4. Algebraic Number Theory, Serge Lang, Springer.
5. The Theory of Algebraic Number Fields, David Hilbert, Springer.

MTH-E1010 – Riemannian Geometry

Unit-I

Riemannian manifold: Riemannian metric, Riemannian manifolds, Partition of unity, Affine connection, Riemannian connection, Riemannian curvature, Scalar curvature, Ricci curvature, Bianchi identities, First fundamental form, An idea of Model spaces.

Unit-II

Submanifolds of Riemannian manifold: Distribution on manifolds, Submanifolds of Riemannian manifold, Hypersurfaces, Second fundamental form, Gauss and Weingarten formulae, Equation of Gauss, Codazzi and Ricci.

Unit-III

Structures on manifolds: Complex manifolds, Almost complex manifolds, Nejenhuis tensor and integrability of structure, Hermitian manifold, Kaehler and nearly Kaehler manifold, Contact structure, Sasakian manifolds.

Unit-IV

Submanifolds of complex and contact manifolds: Submanifolds of Hermitian almost manifolds, Invariant and anti-invariant distributions, Cauchy-Riemann submanifolds, Slant, semi-slant, bislant submanifolds, Quarternionic submanifolds.

Recommended books:

1. Rieamannian geometry, P. Petersen, Graduate Texts in Mathematics, Springer.
2. Riemannian geometry, M. P. do Carmo, Birkhäuser, Boston, Basel, Berlin.
3. Riemannian Geometry - An introduction to curvature, J. M. Lee, Graduate Texts in Mathematics, Springer.
4. Structures on Manifolds, K. Yano and M. Kon, World Scientific Press.
5. Geometry of Submanifolds, B.-Y Chen, Dover Publications.

MTH-E1011 – Topics in Non-Commutative Algebra

Unit-I

Review of rings, modules and algebras. Finite dimension division algebras, simple rings and modules, Endomorphism rings and Schur's Lemma. Central algebras and simple algebras, Skolem-Noether Theorem, Wedderburn theorem on finite division rings, nilpotent ideals, prime and semiprime rings and algebras. Minimal left ideals and Wedderburn's structure theorem.

Unit-II

Semisimple modules and rings, Wedderburn structure theorem for semisimple rings, uniqueness theorem for semisimple rings, structure theorem for Simple Artinian rings, Jacobson radical and its properties, Nakayama Lemma and its applications.

Unit-III

Tensor product of vector spaces, Linear (In)dependence in Tensor products, Tensor product of algebras, Centralizers in Tensor products, Simplicity of Tensor products. Double centralizer theorem, the Brauer group.

Unit-IV

Primitive rings, The Jacobson Density theorem and its alternative versions, primitive rings having minimal left ideals, primitive ideals, structure theorem for primitive rings.

Recommended books:

1. Introduction to non-commutative algebra, Matej Bresar, Springer.
2. Non-commutative algebra, Benson Frab and R. Keith Dennis, Springer.
3. A first course on non-commutative rings, T. Y. Lam, Springer.

MTH-E1012 – Special Functions

Unit-I

Review of Infinite products: definition, necessary condition for convergence, associated series of logarithms, absolute and uniform convergence of infinite products. The Euler or Mascheroni constant γ and the Weierstrass's definition of the Gamma function Γ. A Series for $\frac{\Gamma'(z)}{\Gamma(z)}$. Euler's integral for $\Gamma(z)$. The Beta function. The Factorial function. Legendre's duplication formula. Gauss's multiplication theorem. The behaviour of $\log\Gamma(z)$ for large $|z|$.

Unit-II

The Hypergeometric function: a simple integral representation of the function $F(a, b, c; z)$. $F(a, b, c; 1)$ as a function of the parameters. Evaluation of $F(a, b, c; 1)$. The contiguous function relations. The hypergeometric differential equation, logarithmic solutions of the hypergeometric equation. $F(a, b, c; z)$ as a function of its parameters. Elementary series manipulations. Simple transformations. Relation between functions of z and $1 - z$. Quadratic transformations, theorem due to Kummer.

Unit-III

Orthogonal Polynomials: Simple sets of polynomials, orthogonality, an equivalent condition for orthogonality. Zeros of orthogonal polynomials. Expansion of polynomials. The three-term recurrence relation. The Christoffel-Darboux formula. Normalization, Bessel's inequality. Legendre, Hermite, Laguerre And Jacobi Polynomials.

Unit-IV

Generating Functions: The generating function concept. Bessel Functions: definition of $J_n(z)$, Bessel's differential equation, Bessel's integral. Modified Bessel functions. Elliptic Functions: doubly periodic functions, elementary properties, order of an elliptic function. The Weierstrass function $\mathcal{P}(z)$. Other elliptic functions. A differential equation for $\mathcal{P}(z)$. Connection with elliptic integrals.

Recommended books:

1. Special Functions, Earl D. Rainville, The Macmillan company - New York.
2. Special Functions & Their Applications, N. N. Lebedev, Dover Books on Mathematics.
3. Special Functions and Orthogonal Polynomials, Refaat El Attar, Lulu.com.

MTH-E1013 – Non-Linear Analysis

Unit-I

Compactness in Metric spaces, measure of non-compactness, normed spaces, Banach spaces, Hilbert spaces, uniformly convex, strictly convex and reflexive Banach spaces, Lipchitizian and contraction mapping, Banach's contraction principle, Application to Volterra and Fredholm integral equations.

Unit-II

Nonexpansive, asymtotically nonexpansive, accritive and quasinonexpansive mappings, Fixed point theorem for nonexpansive mappings, nonexpoansive operators in Banach spaces satisfying Opial's condition, The demiclosedness principle.

Unit-III

Schauder's fixed point theorem, condensing maps, fixed points for condensing maps. The modules of convexity and normal structure, radial retraction, Sadoskii's fixed point theorem, set valued mappings.

Unit-IV

Fixed point iterative procedures, The Mann iteration, Lipchitizian and pseudocontractive operators in Hilbert spaces, strongly pseudocontractive operators in Banach spaces, The Ishikawa iterationm stability of fixed point iteration procedures.

Recommended books:

1. V. Berinde, Iterative Approximation of Fixed Points, Lecture Notes in Mathematics, No. 1912, Springer, 2007.

2. M. A. Khansi and W. A. Krik, An Introduction to Metric Spaces and Fixed Point Theory, John-Wiley and Sons, New York, 2001.

3. Sankatha P. Singh, B. Watson and P. Srivastava, Fixed Point Theory and Best Approximation: The KKM-map Principle, Kluwer Academic Publishers Dordrecht, The Netherlands, 1997.

4. V. I. Istratescu, Fixed Point Theory, An Introduction, D. Reidel Publishing Co., 1981.

5. K. Goebel and W. A. Kirk, Topics in Metric Fixed Point Theory, Cambridge University Press, 1990.

MTH-E1014 – Harminic mappings in a plane and Univalent functions

Unit-I

Planar Harmonic Mappings: Differential operators $\frac{\partial}{\partial z}$ and $\frac{\partial}{\partial \bar{z}}$, Composition Rule for Harmonic Mappings, Conformal Mappings, Canonical Representation, Argument Principle for Harmonic Mappings, Critical Points of Harmonic Mappings, Jacobian and Local Univalence

Unit-II

Lewy's Theorem, Solution of Elliptic Partial Differential Equation, Subordination Principle, Schwarz Lemma for Harmonic Mappings, Quasiconformal Diffeomorphism, Conformal Modulus, Basic Properties of Quasiconformal Mappings, Heinz Lemma, Rado's Theorem.

Unit-III

Basic Theory of Univalent Functions, Examples of Functions in the class S, Koebe Function, Disk Automorphism, Range Transformation, Omitted Value Transformation, Bieberbach theorem and its applications, The Area Theorem and its Consequences, The growth and Distortion Theorems.

Unit-IV

Convex and Starlike Functions, Close to Convex Functions, Subclasses of starlike and convex functions and their generalizations, Functions with positive real part, Jack's Lemma, Angular Limit, Angular Derivative.

Recommended books:

1. P.L. Duren, Harmonic Mappings in the Plane, Cambridge Univ.Press, Cambridge, 2004.
2. D. Bshouty and A. Lyzzaik, Problems and Conjectures in Planar Harmonic Mappings, J. Analysis,
3. A. W. Goodman, Univalent Functions I and II, Mariner, Florida, 1983.
4. P. Duren, Univalent Functions, Springer, New York, 1983.
5. Ch. Pommerenke, Univalent Functions, Van den Hoek and Ruprecht, Gottingen, 1975 Volume 18 (2010), 69-81.