CURRICULUM TRANSACTIONAL STRATEGY

MMT-C 204: Differential Geometry

Prerequisites: Multivariable Calculus

COURSE OBJECTIVES

- To get introduced to the concept of a regular parameterized curve in $\mathbb{R}^n$
- To Understand the concept of curvature of a space curve and signed curvature of a plane curve.
- To be able to understand the fundamental theorem for plane curves.
- To get introduced to the notion of Serret-Frenet frame for space curves and the involutes and evolutes of space curves with the help of examples.
- To be able to compute the curvature and torsion of space curves.
- To be able to understand the fundamental theorem for space curves.
- To get introduced to the concept of a parameterized surface with the help of examples.
- To Understand the idea of orientable/non-orientable surfaces.
- To get introduced to the idea of first fundamental form/metric of a surface.
- To Understand the normal curvature of a surface, its connection with the first and second fundamental form and Euler's theorem
- To Understand the Weingarton Equations, mean curvature and Gaussian curvature.
- To understand surfaces of revolution with constant negative and positive Gaussian curvature.
- To understand the isometry between two surfaces and characterization of local isometry between them.
- To be introduced to Christoffel symbols and their expression in terms of metric coefficients and their derivatives.
- To prove Theorema Egregium of Gauss.
- To Discuss the fundamental Theorem for regular surfaces.
- To get introduced to geodesics on a surface and their characterization.
- To understand geodesics as distance minimizing curves on surfaces.
- To find geodesics on various surfaces.
- To Discuss Gauss Bonnet theorem and its implication for a geodesic triangle

COURSE OUTLINE

UNIT I

- The definition of a regular parametrised curve with examples.
- The concept of curvature of a space curve and signed curvature of a plane curve
- The fundamental theorem for plane curves.

- The Serret-Frenet frame for space curves and the notion of torsion of a space curve.
- The involutes and evolutes of space curves

UNIT II

- The concept of a regular parametrised surface with the help of examples.
- The tangent plane at a point on a surface.
- The surface normal and the idea of an orientable surface.
- The first and second fundamental forms.
- The differentiable mapping between regular surfaces and their differential.

UNIT III

- Curvature of a Surface
- Normal curvature and Euler's theorem.
- The Weingarton Equations. Gauss map and its differential.
- The Normal curvature and the second fundamental form. Meusnier's theorem.
- Gaussian curvature: The surfaces with constant positive/Negative Gaussian curvature.
- Lines of Curvature and Rodrigue's formula
- Characterisation of local Isometry between Regular Surfaces.

UNIT IV

- Christoffel Symbols and their expression in terms of metric coefficients and their derivatives.
- Theorema Egregium of Gauss. Invariance of Gaussian curvature under parametric transformation.
- Mainnardi –Codazzi equations and Fundamental theorem for regular surfaces.
- Geodesics: Equations of geodesics, Geodesics on sphere and pseudosphere.
- Geodesics as distance minimizing curves.
- Gauss Bonnet theorem and its implications for a geodesic triangle on a surface.

Classroom Transaction

Unit	Topic	Activity	No. of Tutorials	No. of lectures
I	The definition of a regular parametrised curve with examples.	Assignment	02	02
	The concept of curvature of a space curve and signed curvature of a plane curve	Assignment	01	02
	The fundamental theorem for plane curves.	Assignment	02	02
	The Serret-Frenet frame for space curves and the notion of torsion of a space curve.	Assignment and Presentation	02	01
	The involutes and evolutes of space curves	Assignment and Presentation	04	05

Unit	Topic	Activity	No. of Tutorials	No. of lectures
II	The concept of a regular parametrised surface with the help of examples.	Assignment	02	01
	The tangent plane at a point on a surface	Assignment	01	01
	The surface normal and the idea of an orientable surface.	Assignment and Presentation	02	03
	The first and second fundamental forms.	Assignment	03	04
	The differentiable mapping between regular surfaces and their differential.	Assignment	02	01
	Invariance of area under change of coordinates.			01

Unit	Topic	Activity	No. of Tutorials	No. of lectures
III	Curvature of a Surface .	Assignment	02	01
	Normal curvature and Euler's theorem.	Assignment	02	02
	The Weingarton Equations. Gauss map and its differential.	Assignment and Presentation	02	04
	The Normal curvature and the second fundamental form. Meusnier's theorem.	Assignment	02	03
	Gaussian curvature: The surfaces with constant positive/Negative Gaussian curvature.	Assignment	01	02
	Lines of Curvature and Rodrigue's formula	--	01	02
	Characterisation of local Isometry between Regular Surfaces.		01	01

Unit	Topic	Activity	No. of Tutorials	No. of lectures
IV	Christoffel Symbols and their expression in terms of metric coefficients and their derivatives.	Assignment	02	02
	Theorema Egregium of Gauss. Invariance of Gaussian curvature under parametric transformation.	Assignment	01	03
	Mainnardi –Codazzi equations and Fundamental theorem for regular surfaces.	Assignment	01	02
	Geodesics: Equations of geodesics, Geodesics on sphere and pseudosphere.	--	02	05
	Geodesics as distance minimizing curves.	Assignment and Presentation	01	02
	Gauss Bonnet theorem and its implications for a geodesic triangle on a surface.			02

Text books:

1. John Mc Cleary: Geometry from a differentiable Viewpoint. (Cambridge Univ. Press).
2. Andrew Pressly, Elementary Differential Geometry (Springer Verlag, UTM).
3. Barret O'Neil, Elementary Differential Geometry, Academic Press (2006).
4. C.Baer, Elementary Differential Geometry, Cambridge Univ. Press (2010).

Reference Books:

1. W. Klingenberg: A course in Differential Geometry (Springer Verlag)
2. J. M. Lee : Riemannian Manifolds, An Introduction to Curvature (Springer Verlag)