CURRICULUM TRANSACTIONAL STRATEGY

MMT-E 406: Number Theory-II

Pre-requisites: Advanced number theory.

COURSE OBJECTIVES

- To understand the difference between quadratic residues and non-residues.
- To understand the Legendre symbols and its basic properties.
- To verify the given number is a quadratic residue or non-residue by using Euler's criterion.
- To understand Quadratic reciprocity law.
- To differentiate Jacobi symbol with Legendre ones.
- To understand the definition of Arithmetic functions and multiplicative functions.
- How to find the number of divisors of any given number.
- How to find the sum of divisors of any given number.
- To understand Moebius function and Moebius inversion formula.
- How to characterize even perfect numbers.
- To understand the Farey fractions and rational approximations.
- To differentiate between simple finite continued fractions and infinite continued fractions.
- To approximate irrationals with the help of infinite continued fractions.
- To understand Hurwitz theorem and the best possible constant in Hurwitz theorem.
- To have the basic knowledge about Riemann Zeta functions and the set of primes.
- To understand the definition about Characters
- To understand Dirchlet's Theorem on infinitude of primes in arithmetic progression.
- To understand division algorithm for polynomials and irreducible polynomials.
- How to use Guass Lemma and Schonemann-Eisenstien Criterion for irreducible polynomials.
- To understand algebraic numbers and algebraic integers.
- To understand Quadratic fields and primes in quadratic fields.
- To understand unique factorization property of quadratic fields and some theorems.
- To find the solutions of $x^2 + y^2 = z^2$and $x^3 + y^3 = z^3$ in rational integers.

COURSE OUTLINE

UNIT I

- Quadratic residues and non-residues.
- Legendre symbols and its properties.
- Euler's criterion and Guass's Lemma.
- Quadratic reciprocity law and its applications
- Jacobi symbol; its properties and applications.

UNIT II

- Arithmetic functions, multiplicative functions.
- Moebius function and Moebius inversion formula.
- Perfect numbers and characterization of even perfect numbers.
- Euler's constant and related theorems.
- Farey fractions and rational approximations

UNIT III

- Simple continued fractions.
- Infinite continued fractions and its application to the approximation of irrationals
- Hurwitz theorem and the best possible constant in Hurwitz theorem.
- Riemann Zeta functions and the set of primes.
- Characters and some important relation of characters.
- Dirchlet's Theorem on infinitude of primes in arithmetic progression.

UNIT IV

- Division algorithm for polynomials and irreducible polynomials.
- Guass Lemma and Schonemann-Eisenstien Criterion for irreducible polynomials.
- Algebraic numbers and algebraic integers.
- Quadratic fields and primes in quadratic fields.
- Unique factorization property of quadratic fields and some theorems.

Classroom Transaction

Unit	Topic	Activity	No. of Tutorials	No. of lectures
I	Definitions of Quadratic residues and non-residues	Assignment	02	02
	Legendre symbols and its properties	Assignment	02	03
	Euler's criterion and Guass's Lemma	Assignment and Presentation	02	02
	Quadratic reciprocity law and its applications	Assignment	02	04
	Jacobi symbol; its properties and related results	Assignment	03	04

Unit	Topic	Activity	No. of Tutorials	No. of lectures
II	Definitions of arithmetic functions, multiplicative functions and illustration with examples	Assignment	01	03
	Moebius function and Moebius inversion formula	Assignment	03	04
	Perfect numbers and characterization of even perfect numbers	Assignment	01	02
	Euler's constant and related theorems	Assignment	01	03
	Farey fractions and rational approximations	Assignment and Presentation	02	04

Unit	Topic	Activity	No. of Tutorials	No. of lectures

III	Definition of simple continued fractions with examples: related results	Assignment	02	05
	Infinite continued fractions and its application to the approximation of irrationals	Assignment	02	03
	Hurwitz theorem	Assignment and Presentation	01	02
	Riemann Zeta functions and the set of primes	Assignment	02	03
	Definition of characters and the related theorems	Assignment	02	03
	Dirchlet's Theorem on infinitude of primes in arithmetic progression	--	01	02

Unit	**Topic**	**Activity**	**No. of Tutorials**	**No. of lectures**
IV	Division algorithm for polynomials and irreducible polynomials	Assignment	01	02
	Guass Lemma and Schonemann-Eisenstien Criterion for irreducible polynomials	Assignment	01	04
	Definition of algebraic numbers and algebraic integers with examples	-	01	02
	Quadratic fields and primes in quadratic fields	Assignment	01	04
	Unique factorization property of quadratic fields and some theorems	Assignment and Presentation	03	03

Text Books:

1. W. J. Leveque, Topics in Number Theory Vol. I and II, Addison Wesley publishing House
2. I. Niven, H. S. Zuckerman and H.L. Montgomery, An introduction to the theory of numbers; John Wiley & Sons Inc.
3. David M. Burton, Elementary number theory; Tata Mc-Graw Hill Publishing Company Ltd.
4.George E. Andrews, Number theory; Dover Publications, Inc., New York.