

# CURRICULUM TRANSACTIONAL STRATEGY

## MMT-E408: Commutative Algebra

**Prerequisites:** Basic Knowledge of Groups and Rings

### **COURSE OBJECTIVES**

- To Understand and appreciate the fundamental nature of the ideals, prime ideals and maximal ideals in rings.
- To Understand and develop manipulation skills in the use of operations on submodules e.g., taking sums, products, intersections and annihilators of submodules.
- To understand the idea of exact sequences of modules and their homomorphisms.
- To get introduced the idea of tensor product of modules.
- To get introduced to the idea of rings and modules of fractions.
- To understand the relationship between ideals and submodules with ideals and submodules in ring their corresponding rings and modules of reactions.
- To understand the concept of localization of a ring.
- To get introduced to the concept of primary ideals and primary decomposition of ideals.
- To get introduced to the concept of chain conditions in rings and modules.
- To understand the importance of chain conditions while studying structures of rings and modules and the properties being preserved under the taking of chain conditions.
- To Understand the Hilbert's Basis Theorem.
- To Understand the Structure Theorem for Artin rings.

### **COURSE OUTLINE**

#### **UNIT I**

- Rings and ring homomorphisms, ideals and quotient rings.
- Prime and maximal ideal, nil radical and Jacobson radical.
- Modules, submodules, module homomorphisms and quotient modules.
- Operations on submodules, direct sum and product.
- Finitely generated Modules and Nakayama's Lemma.

#### **UNIT II**

- Exact and Short exact sequences, Snake Lemma and Short five Lemma.
- Tensor product of modules, basic properties and flat modules.
- Restriction and extension of scalars.
- Algebras and their tensor product.
- Rings and modules of fractions, local properties, extended and contracted ideals.

#### **UNIT III**

- Primary ideals and primary decomposition of ideals.

- First and 2<sup>nd</sup> uniqueness theorems.
- Integral independence and closeness of Integral domains, Going up and Going down theorems.
- Valuation rings and Noether's normalization lemma.

#### UNIT IV

- Chain conditions, Noetherian and Artinian rings and modules and Hilbert's Basis theorem.
- Primary decomposition in Noetherian rings.
- Artin rings and structure theorem for Artin rings.
- Discrete valuation rings and Dedekind domains.

### Classroom Transaction

Unit	Topic	Activity	No. of Tutorials	No. of lectures
<b>I</b>	Rings and ring homomorphisms, ideals and quotient rings.	Assignment	01	02
	Prime and maximal ideal, nil radical and Jacobson radical.	Assignment	01	02
	Modules, submodules, module homomorphisms and quotient modules.	Assignment	01	02
	Operations on submodules, direct sum and product.	Assignment and Presentation	02	02
	Finitely generated Modules and Nakayama's Lemma.	Assignment	02	02
Unit	Topic	Activity	No. of Tutorials	No. of lectures
<b>II</b>	Exact and Short exact sequences, Snake Lemma and Short five Lemma.	Assignment	01	03
	Tensor product of modules, basic properties and flat modules.	Assignment and Presentation	03	02
	Restriction and extension of scalars.	Assignment	01	01

	Algebras and their tensor product	Assignment	01	01
	Rings and modules of fractions, local properties, extended and contracted ideals	Assignment	02	02

Unit	Topic	Activity	No. of Tutorials	No. of lectures
<b>III</b>	Primary ideals and primary decomposition of ideals	Assignment	02	02
	First and 2 <sup>nd</sup> uniqueness theorems.	Assignment	02	01
	Integral independence and closeness of Integral domains, Going up and Going down theorems.	Assignment and Presentation	02	04
	Valuation rings and Noether's normalization lemma.	Assignment	02	02

Unit	Topic	Activity	No. of Tutorials	No. of lectures
<b>IV</b>	Chain conditions, Noetherian and Artinian rings and modules and Hilbert's Basis theorem.	Assignment	02	04
	Primary decomposition in Noetherian rings	Assignment	01	02
	Artin rings and structure theorem for Artin rings.	Assignment	01	02
	Discrete valuation rings and Dedekind domains.	Assignment	02	02

**Text Books:**

1. *Introduction to Commutative Algebra* M.F. Atiyah & I. G. Macdonald, Addison Wesley publishing company.
2. *Commutative Algebra I, II* Zariski Oscar and Samuel Pierre, Springer

**Reference Books:**

1. *Algebra* Serge Lang Springer, 3rd Edition.
2. *Exercises in Rings and Modules*, Lam Springer.