**Session 2019-2020**

**Integrated Bsc-Msc Mathematics**
**Revised syllabus of VII & VIII Semester**

## Semester-VII

| Subject Code | Subject | Contact Hours per Week (Theory+Tutorials+Presentations) | Credits | Marks | |
|---|---|---|---|---|---|
| | | | | CIA | ESE |
| MTH-701 | Analysis –V | [3+2+1] | 4 | 40 | 60 |
| MTH-702 | Algebra –V | [3+2+1] | 4 | 40 | 60 |
| MTH-703. | Advanced Complex Analysis | [3+2+1] | 4 | 40 | 60 |
| MTH-704 | Advanced Number Theory | [3+2+1] | 4 | 40 | 60 |
| MTH-705 | Operations Research-II | [3+2+1] | 4 | 40 | 60 |
| SS | Soft Skill Elective | [4+0+0] | 4 | 40 | 60 |
| | **Semester Credits** | **[19+10+5]** | **24** | **240** | **360** |
| | | | | **600** | |
| | **Subtotal** | | **168 of 240** | **4200 of 6000** | |

## Semester-VIII

| Subject Code | Subject | Contact Hours per Week (Theory+Tutorials+Presentations) | Credits | Marks | |
|---|---|---|---|---|---|
| | | | | CIA | ESE |
| MTH-801 | Fields and Galois Theory | [3+2+1] | 4 | 40 | 60 |
| MTH-802 | Topology-II | [3+2+1] | 4 | 40 | 60 |
| MTH-803 | Fourier Analysis | [3+2+1] | 4 | 40 | 60 |
| MTH-804 | Calculus of Variations and Integral Equations | [3+2+1] | 4 | 40 | 60 |
| MTH-805 | Topics in Graph Theory | [3+2+1] | 4 | 40 | 60 |
| SO | Social Orientation Elective | [4+0+0] | 4 | 40 | 60 |
| | **Semester Credits/Marks** | **[19+10+5]** | **24** | **240** | **360** |
| | | | | **600** | |
| | **Subtotal** | | **192 of 240** | **4800 of 6000** | |

**Semester-VII**

**MTH-701: Analysis -V**

**Unit-I**
Functions of bounded variations and monotone function, relationship between them. Continuous function and functions of bounded variations, necessary and sufficient condition for a function to be of bounded variation. Algebra of space of bounded variation function. Absolute continuous functions and its relation with bounded variation functions. Lipchitz functions and its relations with function of bounded variations.

**Unit-II**
Absolute continuity and bounded variations, their relationship and counter examples, indefinite integral of L-integrable functions and its absolute continuity, necessary and sufficient conditions for bounded variation, Vitali Covering Lemma and a.e. differentiability of monotone function f and $\int_a^b f' \le f(b) - f(a)$.

**Unit-III**
Lebesgue measure on $R^n$. Product measure and iterated integrals, examples of non-itegrable functions whose iterated integrals exist and equal, Fubbini Theoren and Tonelli Theorem. Differentiation and integration.

**Unit-IV**
The Randon Nikodym Theorem and its applications: Absolutely continuous measures and the Randon Nikodym theorem, Computation Randon Nikodym derivative, Change of variable formulae.

**Text Books**

1. *Principles of Real Analysis*, H. L. Royden, PHI.
2. *Principles of Real Analysis*, C. D. Aliprantis and O. Burkinshow, Academic Press, New York.

***References:***

1. *Measure and Integration*, I K Raina, Narosa.
2. *Infinite Dimensional Analysis*, C. D. Aliprantis and K C Border, Springer.

## MTH-702: Algebra-V

**Unit-I:**

Finite abelian Groups: Direct products and semi direct products of groups, direct sums and its properties, divisible and torsion groups. Free abelian groups, p-primary groups and primary decomposition theorem. Bases Theorem, Fundamental Theorem for finite abelian groups and its applications.

**Unit-II:**

Free groups and presentations: definition, examples and properties, Basis of a free group, and its properties, rank of a free group. Generator, relations, and presentations of free groups with examples and properties. Nielsen-Schreier Theorem on subgroups of free groups.

**Unit-III**

Review of modules and their properties, simple modules, Exact and short exact sequences, split sequences, Four Lemma, Five Lemma and Short five Lemma, Basics of categories and Functors.

**Unit-IV**

Free modules: properties and applications, Tensor product of Modules with basic properties and applications. Projective, injective and flat modules, Modules with chain conditions: Noetherian and Artinian modules.

**Suggested Books:**


1. J. J. Rotman, Advanced Modern Algebra, second addition, Graduate studies in Mathematics, Vol. 144, AMS.
2. D S Dumit and R M Foote, Abstract Algebra, 3rd Edition, John Wiley.

## MTH-703: Advanced Complex Analysis

**Unit-I**

Maximum Modulus Principle, Schwarz Lemma and its generalization, Meromorphic function, Argument Principle with applications, Rouche's theorem with application, Inverse function Theorem, Poisson integral formula for a circle and half plane, Poisson Jenson formula, Carleman's theorem, Hadmard three-circle theorem and the theorem of Borel and Caratheodory**.**

**Unit-II**

Principle of analytic continuation, uniqueness of direct analytical continuations and uniqueness of analytic continuation along a curve. Power series method of analytic continuation, Functions with natural boundaries and related examples. Shewartz reflection principle, functions with positive real part.

**Unit-III**

Space of analytic functions, Hurwitz's theorem, Montel's theorem, Riemann Mapping theorem, Weistrass factorization theorem, Gamma function and its properties. Riemann Zeta function, Reimann's functional equation.

**Unit IV**

Harmonic functions on a disc, Harnack's inequality and theorem, Drichlet's problem, Green's functions. Canonical products, order of an entire functions, Exponential convergence, Borel theorem, Hadmards factorization theorem, Univalent function. Bieberbach's conjecture (statement only) and the 1/4 – theorem.

**Text Books:**

1. *Complex Analysis,* L. Ahlfors, Springer.
2. *Theory of Functions,* E.C. Titchmarsh Oxford University Press

***<u>References:</u>***

1. *Functions of a complex variable –I,* J.B. Conway, Springer.
2. *Complex Analysis,* Richard Silverman, Dover publications.
3. *Theory of Functions of a Complex variable*, A. I. Markushevish,

## MTH-704: Advanced Number Theory

**Unit I**
Power Residue, Quadratic residue and non-residues, The Legendre symbols, Definition and basic properties, Euler's Criterion, Gauss lemma. Characterization of primes for which 2,-2, 3,-3, 5, 6 and 10 are quadratic residues or non-residues.

**Unit II**
Jacobi symbol and its properties. The law of reciprocity for Jacobi symbols.
Arithmetic functions, multiplicative functions, definition and basic examples, the function [x] and its properties, the symbols "0","0", "~",Euler constant gamma, tau, sigma, phi, the Mobius function, Mobius inversion formula.

**Unit III**
Simple continued fractions, Applications of the theory of infinite continued fractions to the approximations of irrationals by rationals, Hurwitz theorem and its sharpness. Irrationality of e and π, relation between Riemann zeta function and the set of primes, Characters, the orthogonality relation of characters, Dirichlet characters, the Dirichlet L-function and its properties, Dirichlet's theorem.

**Unit IV**
Polynomials, Division Algorithm for polynomials, irreducible polynomials, primitive polynomials, Gauss lemma, Schroeder-Einstein criteria for irreducibility, Algebraic Number and Algebraic integers, minimal polynomial of an algebraic numbers and Algebraic integers, minimal polynomial of an algebraic number Algebraic number field, Primes in Quadratic forms, Euclidean Quadratic forms, unique factorization, EQF has the UFP, solutions of $x^2+y^2 = z^2$ and $x^3+y^3 = z^3$ in rational integers.

**Text Books:**

1. *Topics in number theory*, W. J. Leveque, Vol .I, Addison Wesley publications, company, NIC.
2. *An introduction to the theory of numbers, I*. Niven & H.S Zuckerman.

**References:**

1. *Number theory,* Bevich and I. R. Sharfarich, Academic press.

## MTH-705: Operations Research-II

**Unit I**:
Revised Simplex Method, Column Simplex Method; Column Dual Simplex Method Applications of Duality in LP problems, Forming Rules of a Dual from the primal, Dual Simplex Method, Duality Theorem and Primal Dual Relations,

**Unit II:**
Complementary Slackness Theorem and Complementary Slackness Conditions (CSC), Applications of CSC for finding solution of primal from the solution of the dual and vice versa. Sensitivity Analysis and parametric Programming.

**Unit III:**
Game Theory: Rectangular game, minimax-maximin principle, Two person zero-sum games, pure strategies, saddle point and mixed strategies, Graphical Solution of a 2xn, mx2 games, Principle of Dominance. Equivalence between Game Problem and LPP.

**Unit IV:**
Bounded Variables Techniques, Integer Programming, Goal programming (GP), difference between LP and GP. Model formulation. Graphical solution for Goal Programming, Pre-emptive and Archimedean Goal Programming Methods.

**Text Books:**

1. *Introduction to Mathematical Programming*, Hiller and Lieberman, McGraw Hill Book Co.
2. *Linear Programming and network Flows*, Bazara Jarvis and Sherali, John Wiley.

**References:**

2. *Linear Programming*, Ignizio and Cavalier, Prentice Hall.
3. *Operations Research, Principal and Practice*, Ravindran A., Philips, D.T., and Soleberg, J.J, John Wiley.

## Semester-VIII

## MTH-801: Fields and Galois Theory

**Unit-I**
Field extensions: basic results and examples, degree of extension and related results, finite and algebraic extensions, finitely generated and simple extensions. Classical straightedge and compass constructions.

**Unit-II**
Splitting fields: examples and related results, existence and uniqueness of splitting fields, Algebraic closure and algebraically closed fields, separable and inseparable extensions, prefect fields. Characterization of finite fields, polynomials over finite fields, traces and norms. Cyclotomic polynomials and extensions.

**Unit-II**
Field automorphisms and fixed fields, Galois groups and Galois extensions. Fundamental Theorem of Galois Theory, with applications and related results. Composite extensions, cyclotomic extensions and abelian extensions over Q.

**Unit-IV**
Symmetric and elementary symmetric functions. Discriminant of a polynomial, Galois groups of polynomials of degree 2, 3 and 4. Radical extensions, Solvability by radicals of cubic and quartic equations and Insolvability of the quintic.

**Suggested Texts:**


1. Abstract Algebra by David S. Dummit and Richard M. Foote, 3rd edition, Wiley.

2. Advanced Modern Algebra by Joseph J Rotman, Graduate Studies in Mathematics, Volume 114, American Mathematical Society.

3. Field and Galois Theory by Patrick Morandi, Graduate texts in Mathematics, Springer.

4. Galois Theory by Ian Stewart, 3rd edition, Chapman and Hal CRC.

5. Introduction to Finite Fields and Their Applications by Rudolf Lidl and Harald Niederreiter, Cambridge University Press.

# MTH-802: Topology-II

**Unit I:**

Separation. Urysohn's lemma and Tietze extension theorem. Urysohn Imbedding theorem. Stone-Cech compactification, Connected Spaces. $R^n$ and $C^n$ are connected.

**Unit II:**

Weierstrass approximation theorem. Stone-Weierstrass theorems (real and complex). Locally compact Hausdorff Spaces, extended Stone-Weierstrass theorems.

**Unit III:**

Paths and homotopy, homotopy equivalence. Categories and functors, The fundamental group of the circle. Homotopy lifting lemma, Some applications.

**Unit IV:**

Quotient topology, Group actions and orbit spaces, the van Kampen theorem, covering spaces, Lifting criterion, Universal covers and application to free groups.

**Text books**

1. G. F. Simmons, *Introduction to topology and modern analysis*, Tata Mac Graw-Hill Edition 2004.
2. Allen Hatcher, *Algebraic Topology*. Cambridge University Press

## MTH-803: FOURIER ANALYSIS

**Unit I**

Review of Convergence of series of functions. Even and Odd Functions, Definition of the Fourier series, Fourier Sine Series and Fourier Cosine Series , Calculation of Fourier Coefficients, Convergence of Fourier series, Cesaro Summability, Dirichlet Kernel, Fejer Kernel, Properties of Fejer Kernel , Fejer Theorem.

**Unit II**

Pointwise convergence of Fourier series, Dirichlet's Criterion. Gibb's phenomenon. The $L_2$ theory of Fourier Series: Bessel's Inequality. Convergence of Fourier series. The orthonormal expansions in $L_2[a,b]$. Examples of orthonormal families of functions. Equivalent conditions for a family of orthonormal functions to be Complete.

**Unit III**

One dimensional Wave Equation- The vibrating String, Derivation of Wave Equation, Solution of Wave Equation, D' Alembert Solution of Wave Equation, Laplace Equation in two dimensions, Solution of Laplace Equation, Uniqueness of the Solution of Dirichlet Problem.

**Unit IV**

Introduction to the Fourier Transform- Periodic and Aperiodic Function, Fourier transform, Examples of Fourier transform and their graphical representation, Basic properties of Fourier transform, Linearity, Shifting and Scaling, The Fourier Transform of Derivatives and Integrals Inverse Fourier Transform.

**Reference Books:**

1. Methods of Real Analysis Richard R. Goldberg

2. A Textbook of Real Analysis S.C. Malik

## MTH-804: Calculus of Variation and Integral Equations

**Unit-I**

Calculus of Variations: Introduction, the Brachistochrone problem, Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Calculus of variation verses extrema problems of functions of n variables.

**Unit-II**

Variational methods for boundary value problems in ordinary and partial differential equations. The optimality principle.

**Unit-III**

Linear Integral Equations: Introduction, Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels.

**Unit-IV**

Nonlinear voltra integral equations and applications, Basic elements of collocation methods. Characteristic numbers and eigen functions, resolvent kernel.

**Text Books**

1. *Calculus of variation*, M Gelfand and S V Fomin, Dover.
2. *Integral equations*, F G Tricomi, Dover.

**References:**

1. *Calculus of variations with applications to physics*, R Weinstocks, Dover.
2. *Introduction to non linear differential equations and integral equations*, H L Davis, Dover.

## MTH-805: Topics in Graph Theory

**Unit-I**

Cut vertices and cut edges and their properties. Cut-sets and their properties, vertex connectivity, edge connectivity, Whitney's theorem, Menger's theorem (vertex and edge form), properties of a bond, block graphs,

**Unit II:**

Planar graphs, Kuratowski's two graphs, embedding on a sphere, Euler's formula, Kuratowski's theorem, geometric dual of a planar graph, Whitney's theorem on duality, regular polyhedrons. Number of regular polyhedrons.

**Unit-III**

Graph Matrices: Incidence matrix A(G), modified incidence matrix Af , cycle matrix B(G), fundamental cycle matrix Bf , cut-set matrix C(G), fundamental cut set matrix Cf, relation between Af , Bf and Cf , path matrix, adjacency matrix, matrix tree theorem, Eigen values of adjacency matrix, energy of a graph.

**Unit-IV**

Digraphs, Types of digraphs, types of connectedness, Euler digraphs, Hamiltonian digraphs, arborescence, matrices in digraphs, Camions theorem, tournaments, characterization of score sequences, Landau's theorem, oriented graphs and Avery's theorem, automorphism groups of graphs, graph with a given group, Frucht's theorem, Cayley digraph.

**Text Books:**

1.*Graph Theory*, F. Harary, Addison-Wesley.
2.*Graph Theory with Applications to Engineering and Computer Sciences*, Narsingh Deo, Printice Hall, India Ltd.

**References:**

1. *Introduction to Graph Theory*, D.B. West, PHI.

1. *A First book at Graph Theory,* J. Clark and D.A Holton, World Scientific.
2. *Invitation to Discrete Mathematics*, J.Matousek and J.Nesetril, Oxford University Press.