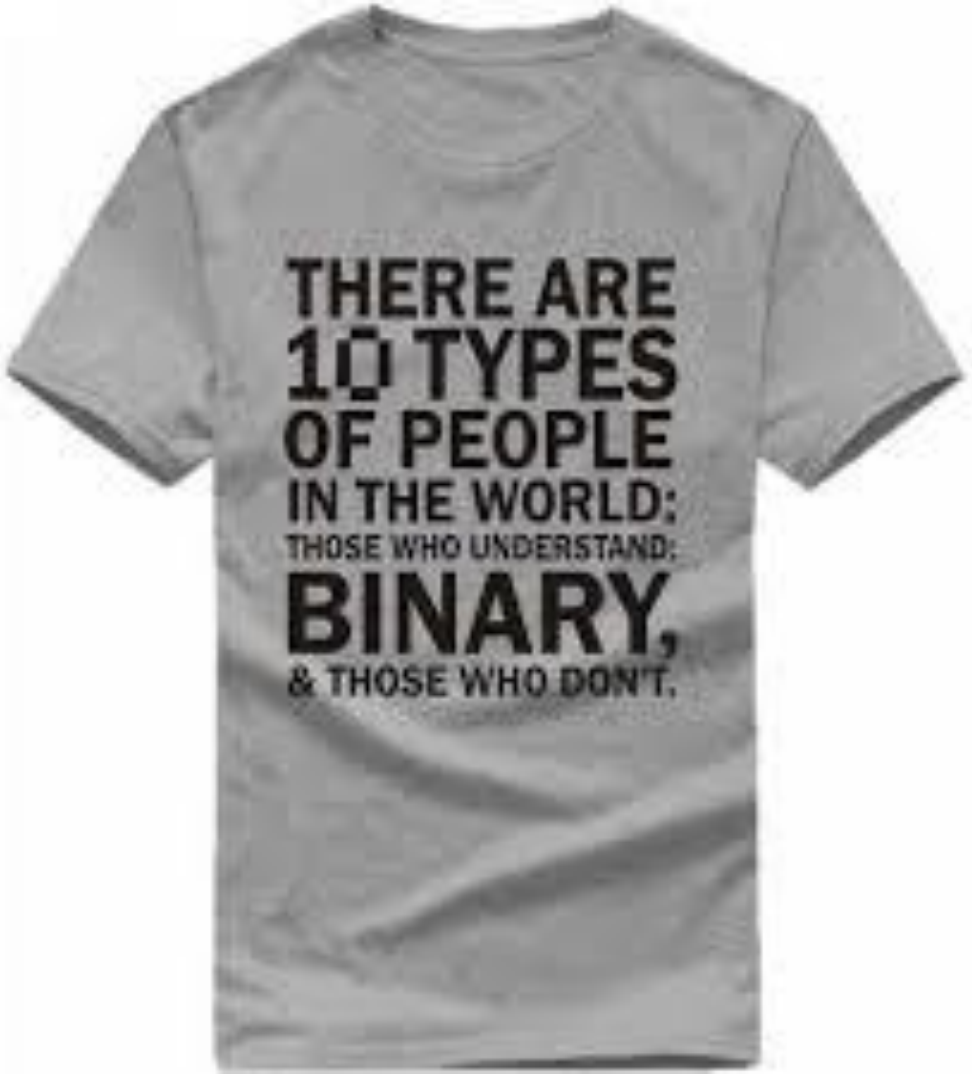
NUMBER SYSTEM & CONVERSION

Number System



Introduction

- Many number systems are in use in digital technology. The most common are :
  - Decimal (Base 10)
  - Binary (Base 2)
  - Octal (Base 8)
  - Hexadecimal (Base 16)

- The ***decimal system*** is the number system that we use everyday

Number System

- **Decimal system** uses 10 symbols ***(digits)***

  0, 1, 2, 3, 4, 5, 6, 7, 8, 9

- **Octal System** uses eight symbols

  0, 1, 2, 3, 4, 5, 6, 7

- **Binary System** uses only two symbols

  0* and *1

- **Hexadecimal System** uses sixteen symbols

  0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

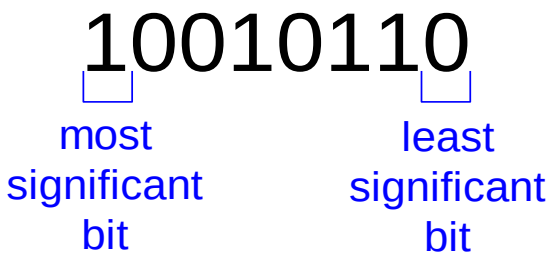
to represent any number, no matter how large or how small.

Familiar System

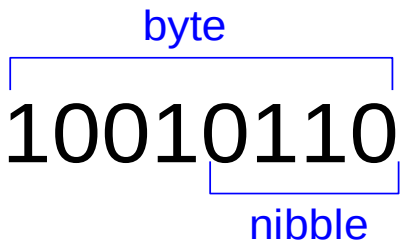
Octal System (Base – 8 ns)	Hexa decimal Number System (Base – 16)	Decimal Number (Base-10)	Binary Equivalent (Base-2)	Base 4 number System (Base -4)
0	0	0	0	0
1	1	1	1	1
2	2	2	10	2
3	3	3	11	3
4	4	4	100	10
5	5	5	101	11
6	6	6	110	12
7	7	7	111	13
10	8	8	1000	20
11	9	9	1001	21
12	A	10	1010	22
13	B	11	1011	23
14	C	12	1100	30
15	D	13	1101	31
16	E	14	1110	32
17	F	15	1111	33

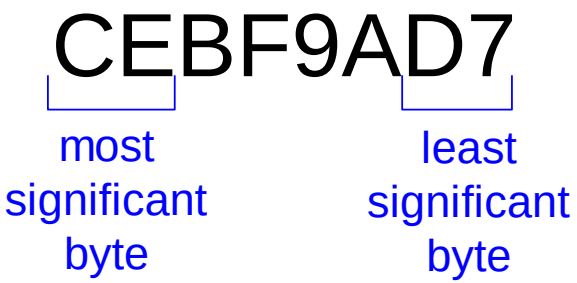
Bits, Bytes, Nibbles

- Bits (b)



- Bytes & Nibbles
  - Byte (B) = 8 bits
    - Used everyday
  - Nibble (N) = 4 bits
    - Not commonly used



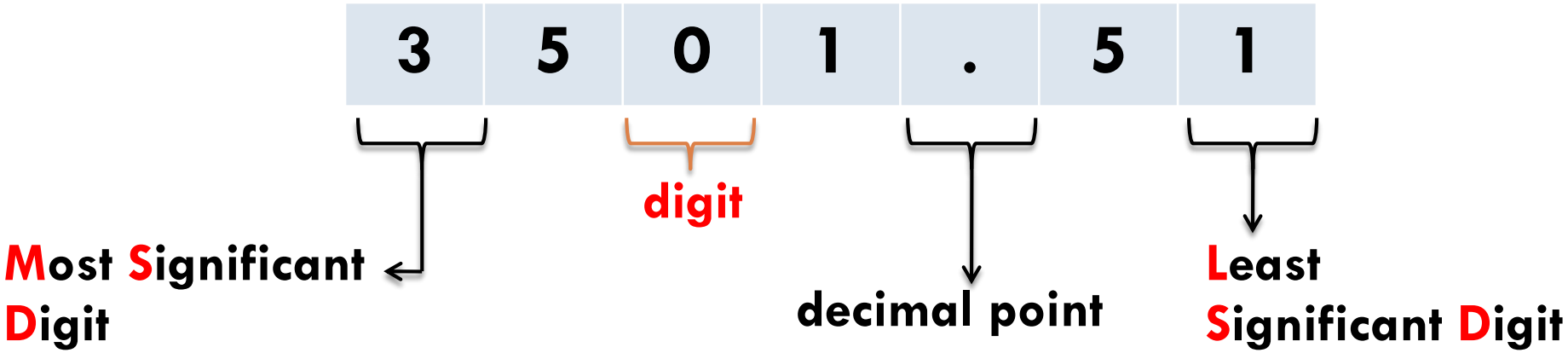


KB, MB, GB …

- In computer, the basic unit is byte (B)
- And, we use KB, MB, GB many many many times
  - 2^{10} = 1024 = **1KB (kilobyte)**
  - 2^{20} = 1024 x 1024 = **1MB (megabyte)**
  - 2^{30} = 1024 x 1024 x 1024 = **1GB (gigabyte)**
- How about these?
  - 2^{40} = **1TB (terabyte)**
  - 2^{50} = **1PB (petabyte)**
  - 2^{60} = **1EB (exabyte)**
  - 2^{70} = **1ZB (zettabyte)**
  - …

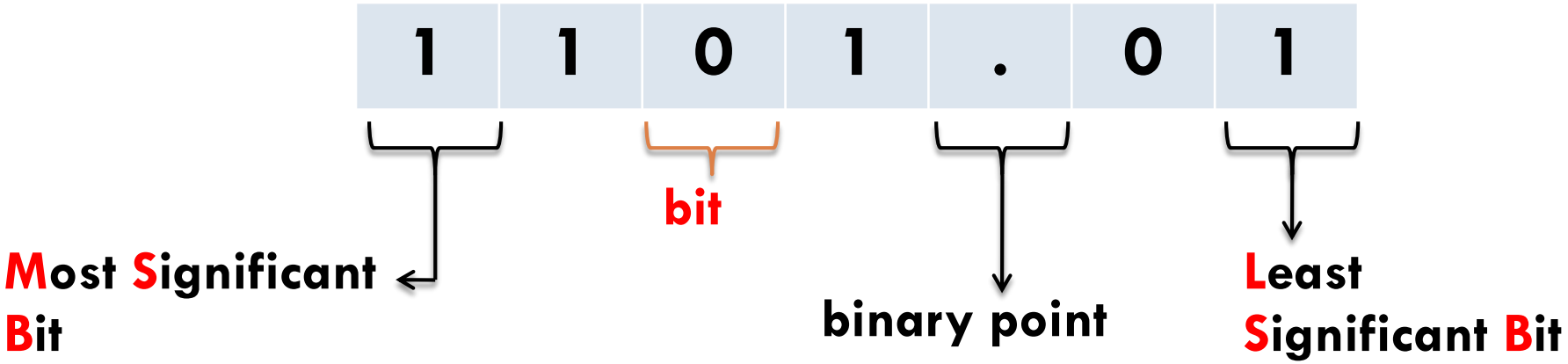
Decimal System

- **The decimal system** is composed of 10 numerals or symbols. These 10 symbols are 0,1,2,3,4,5,6,7,8,9; using these symbols as digits of a number, we can express any quantity.
- **Example : 3501.51**



Binary System

- **The binary system** is composed of 2 numerals or symbols 0 and 1; using these symbols as digits of a number, we can express any quantity.
- **Example : 1101.01**



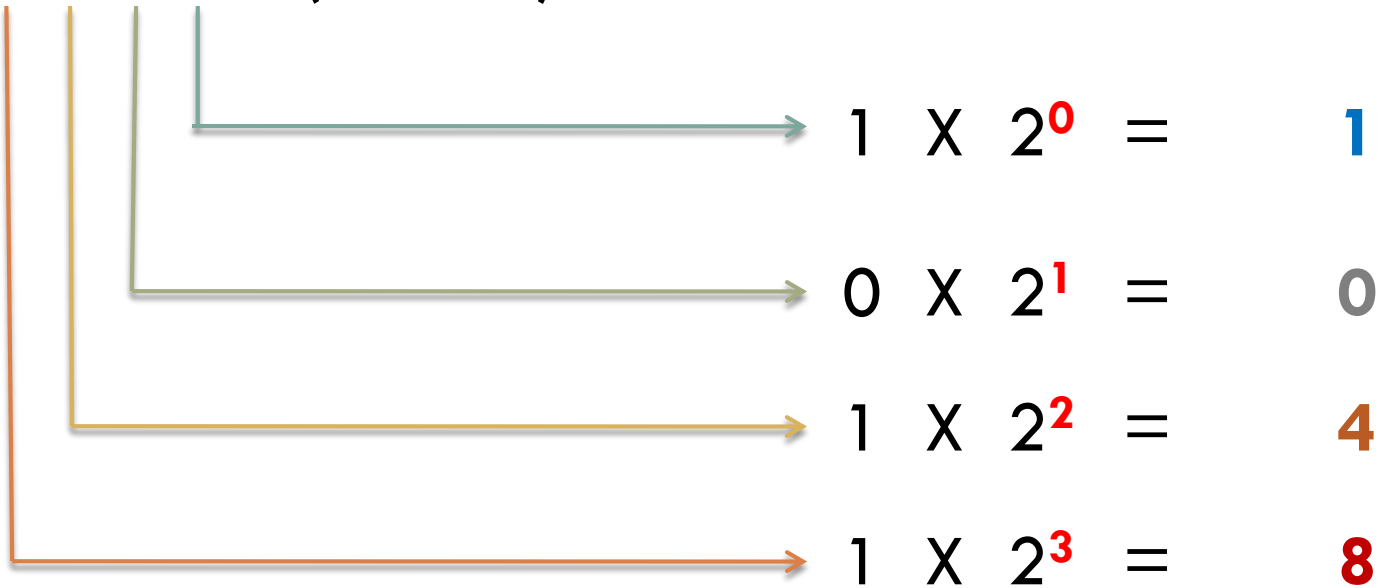
Decimal Number Quantity (positional number)

- 3 5 0 1 (base-10)

3000 + 500 + 0 + 1 = 3501

Binary-to-Decimal Conversion

- 1 1 0 1 (base-2)



$1 \times 2^0 = 1$

$0 \times 2^1 = 0$

$1 \times 2^2 = 4$

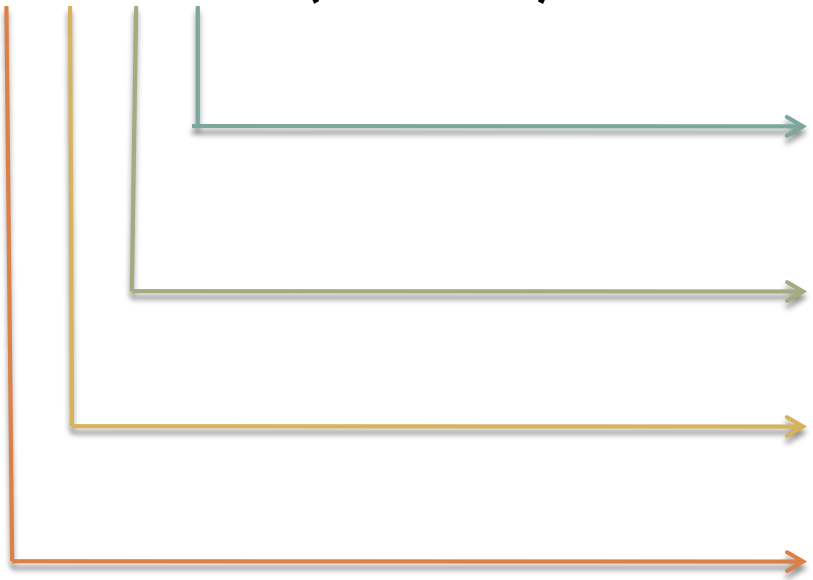
$1 \times 2^3 = 8$

$8 + 4 + 0 + 1 = 13$

$1101_2 = 13_{10}$

Octal-to-Decimal Conversion

- 5 2 1 7 (base-8)



$7 \times 8^0 = 7 \times 1 = 7$

$1 \times 8^1 = 1 \times 8 = 8$

$2 \times 8^2 = 2 \times 64 = 128$

$5 \times 8^3 = 5 \times 512 = 2560$

$2560 + 128 + 8 + 7 = 2703$

$5217_8 = 2703_{10}$

Hexadecimal-to-Decimal Conversion

- 1 A C F (base-16) [A = 10, B = 11, C = 12, D = 13, E = 14, F = 15]

$15 \times 16^0 = 15 \times 1 = 15$

$12 \times 16^1 = 12 \times 16 = 192$

$10 \times 16^2 = 10 \times 256 = 2560$

$1 \times 16^3 = 5 \times 4096 = 20480$

$20480 + 2560 + 192 + 15 = 23247$

$1ACF_{16} = 23247_{10}$

Decimal Number Quantity (fractional number)

- . 5 8 1 (base-10)

$5 \times 10^{-1} = 5 \times 0.1 = 0.5$

$8 \times 10^{-2} = 8 \times 0.01 = 0.08$

$1 \times 10^{-3} = 1 \times 0.001 = 0.001$

$0.5 + 0.08 + 0.001 = 0.581$

Binary-to-Decimal Conversion

- . 1 0 1 (base-2)

$1 \times 2^{-1} = 1 \times 0.5 = \mathbf{0.5}$

$0 \times 2^{-2} = 0 \times 0.25 = 0$

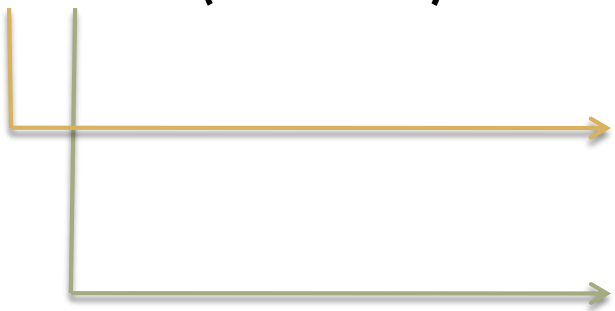
$1 \times 2^{-3} = 1 \times 0.125 = \mathbf{0.125}$

$0.5 + 0 + 0.125 = 0.625$

$0.101_2 = 0.625_{10}$

Octal-to-Decimal Conversion

- . 2 5 (base-8)



$2 \times 8^{-1} = 2 \times 0.125 = \mathbf{0.25}$

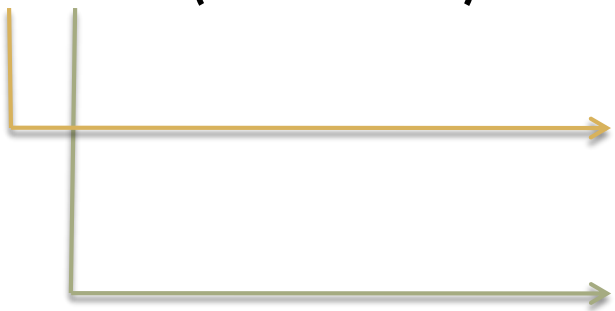
$5 \times 8^{-2} = 5 \times 0.015625 = \mathbf{0.017825}$

$0.25 + 0.017825 = 0.267825$

$0.25_8 = 0.267825_{10}$

Hexadecimal-to-Decimal Conversion

- . F 5 (base-16)



$15 \times 16^{-1} = 15 \times 0.0625 = 0.9375$

$5 \times 16^{-2} = 5 \times 0.00390625 = 0.01953125$

$0.9375 + 0.01953125 = 0.95703125$

$0.F5_{16} = 0.95703125_{10}$

Exercise 1

- Convert these binary system numbers to decimal system numbers

  a) 100101101

  b) 11100.1001

  c)111111

  d)100000.0111

- b)

  $1x2^4 + 1x2^3 + 1x2^2 + 0x2^1 + 0x2^0 + 1x2^{-1} + 0x2^{-2} + 0x2^{-3}\,1 + 1x2^{-4}$

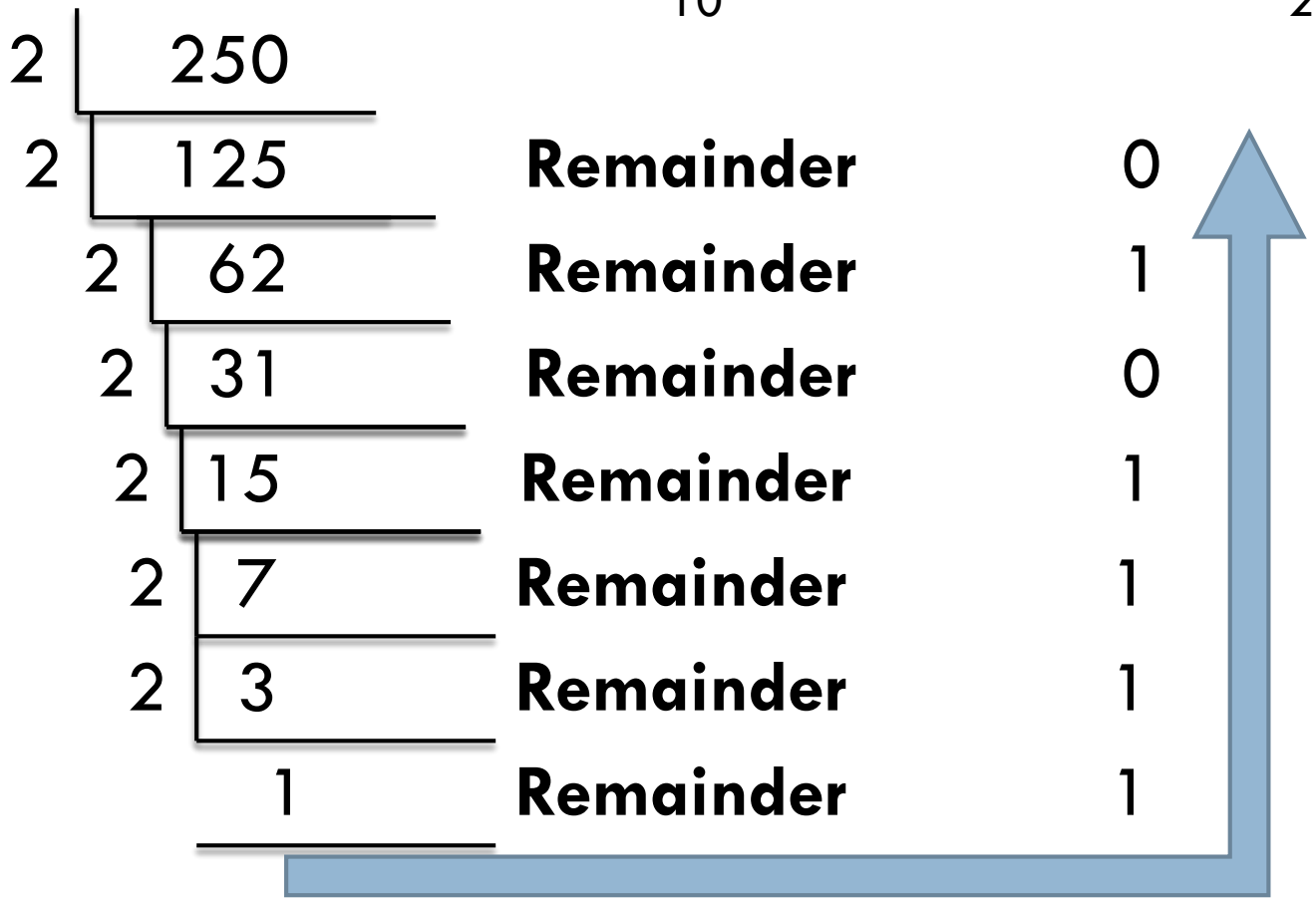
  = 16 + 8 + 4+ 0 + 0 + 0.5 + 0 + 0 + 0.0625

  = 28.5625

Decimal-to-Binary Conversion (positional number)

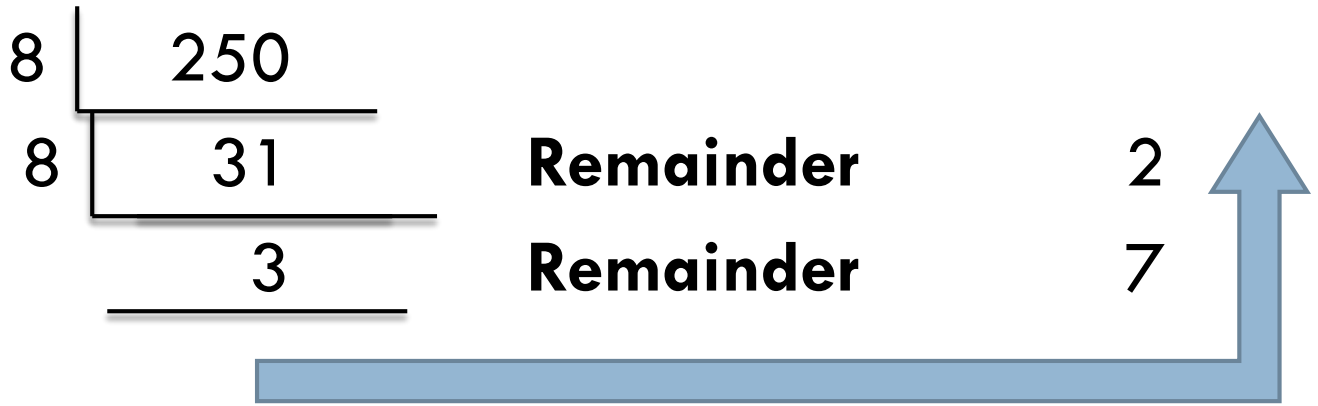
- 2 5 0

$250_{10} = 1\ 1\ 1\ 1\ 1\ 0\ 1\ 0_2$



Decimal-to-Octal Conversion

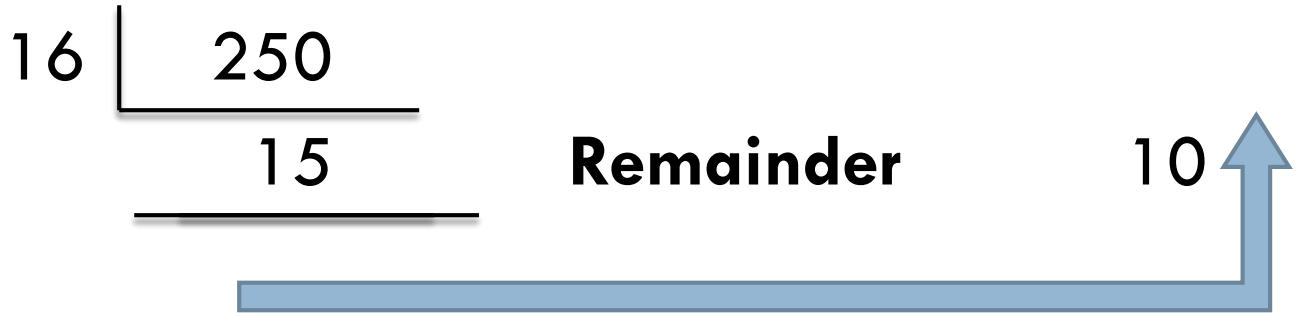
- 2 5 0



$250_{10} = 372_8$

Decimal-to-Hexadecimal Conversion

- 2 5 0



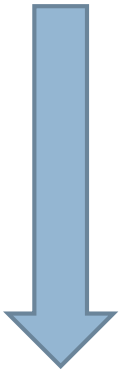
$250_{10} = 15\ \ 10_{16}$?

$= \mathbf{FA_{16}}$

Decimal-to-Binary Conversion (fractional number)

- 0 . 4375

$$
\begin{aligned}
0.4375 \times 2 &= 0.8750 \\
0.8750 \times 2 &= 1.75 \\
0.75 \times 2 &= 1.5 \\
0.5 \times 2 &= 1.0
\end{aligned}
$$



$$0.4375_{10} = 0.0111_{2}$$

Decimal-to-Octal Conversion

- 0 . 4375

$0.4375 \times 8 = 3.5$

$0.5 \times 8 = 4.0$

$$0.4375_{10} = 0.34_{8}$$

Decimal-to-Hexadecimal Conversion

- 0 . 4375

$0.4375 \times 16 = 7.0$ 

$$0.4375_{10} = 0.7_{16}$$

Example :Decimal-to-Binary Conversion (Estimation)

- 0 . 7 8 2

0.782 x 2 = **1.564**
0.564 x 2 = **1.128**
0.128 x 2 = **0.256**
0.256 x 2 = **0.512**
0.512 x 2 = **1.024**
0.024 x 2 = **0.048**
0.048 x 2 = **0.096**
0.192 x 2 = **0.384**
0.384 x 2 = **0.768**
0.768 x 2 = **1.536**

11001_2 → $2^{-1} + 2^{-2} + 2^{-5}$
→ 0.5 + 0.25 +0.03125
→ **0.78125**

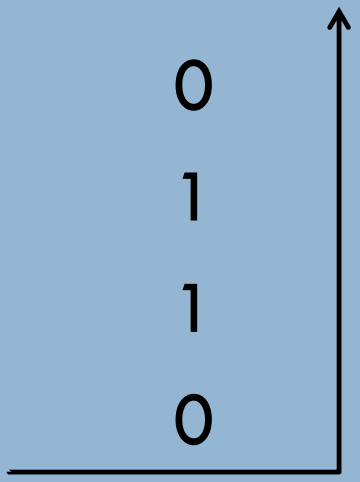
1100100001_2
→ $2^{-1} + 2^{-2} + 2^{-5} + 2^{-10}$
→ 0.5 + 0.25 +0.03125 + 0.0009765625
→ **0.7822265625**

Exercise 2

- Convert these decimal system numbers to binary system numbers
  - a)127
  - b)38
  - c)22.5
  - d) 764.375

c) $(22.5)_{10}=(\ ?\ )_2$

2 | 22		
2 | 11	0	
2 | 5	1	
2 | 2	1	
1	0	



$0.5 \times 2 = \mathbf{1}.0$

$\Rightarrow (22.5)_{10}=(\ 10110.1\ )_2$

Base X – to – Base Y Conversion

- We can convert base x number to base y number by following these steps :
  - Convert base x to base 10 (decimal system number)
  - Then, convert decimal number to base y

Example

- Convert 372.34_8 to hexadecimal system number
  - Convert 372.34_8 to decimal system number
    - $372.34_8 = (3x8^2)+(7x8^1)+(2x8^0) \ . \ (3x8^{-1}) + (4x8^{-2})$
      $= 192 + 56 + 2 \ . \ 0.375 + 0.0625$
      $= 250 \ . \ 4375$
  - Convert 250.4375_{10} to hexadecimal system number
    - 250.4375_{10}

Positional number

250 / 16 = 15 remainder 10
250 → FA_{16}

Fractional number

0.4375 * 16 = 7.0
0.4375 → 0.7_{16}

$372.34_8 = FA.7_{16}$

Exercise 3 (TODO)

- Convert these numbers to octal system number
  - 11100.1001_2
  - 111111_2
  - $5A.B_{16}$
- Convert these numbers to binary system number
  - $5A.B_{16}$
  - 75.2_8

Thank you